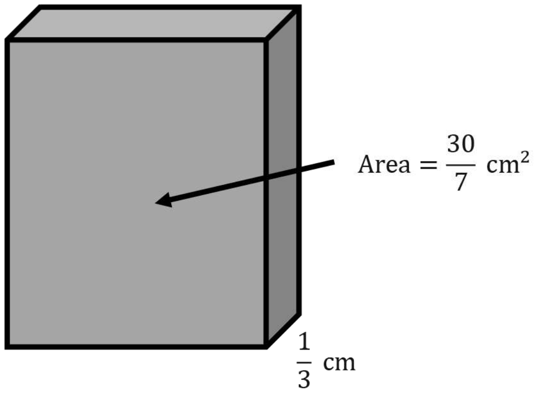
Name ____ Date ____________________

The Formulas for Volume

1. A new company wants to mail out samples of its hair products. The company has a sample box that is a rectangular prism with a rectangular base with an area of $23\frac{1}{3}\ \text{in}^2$. The height of the prism is $1\frac{1}{4}\,\text{in}$.
   Determine the volume of the sample box.

2. A different sample box has a height that is twice as long as the original box described in Problem 1. What is the volume of this sample box? How does the volume of this sample box compare to the volume of the sample box in Problem 1?

1. Determine the volume of the rectangular prism.



2. Determine the volume of the rectangular prism in Problem 1 if the height is quadrupled (multiplied by four). Then, determine the relationship between the volumes in Problem 1 and this prism.

3. The area of the base of a rectangular prism can be represented by B, and the height is represented by h.

   a. Write an equation that represents the volume of the prism.

   b. If the area of the base is doubled, write an equation that represents the volume of the prism.

   c. If the height of the prism is doubled, write an equation that represents the volume of the prism.

   d. Compare the volume in parts (b) and (c). What do you notice about the volumes?

   e. Write an expression for the volume of the prism if both the height and the area of the base are doubled.

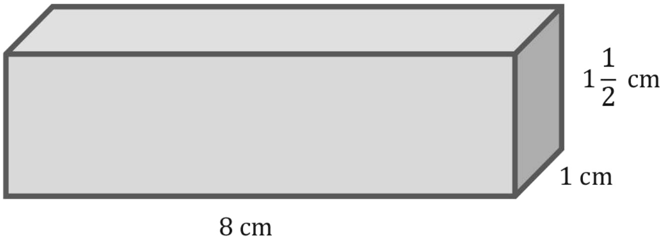
4. Determine the volume of a cube with a side length of $5\frac{1}{3}$ in.

5. Use the information in Problem 4 to answer the following:

   a. Determine the volume of the cube in Problem 4 if all of the side lengths are cut in half.

   b. How could you determine the volume of the cube with the side lengths cut in half using the volume in Problem 4?

6. Use the rectangular prism to answer the following questions.



a. Complete the table.

Length	Volume
$l = 8$ cm	
$\frac{1}{2}l =$	
$\frac{1}{3}l =$	
$\frac{1}{4}l =$	
$2l =$	
$3l =$	
$4l =$	

b. How did the volume change when the length was one-third as long?

c. How did the volume change when the length was tripled?

d. What conclusion can you make about the relationship between the volume and the length?

7. The sum of the volumes of two rectangular prisms, Box A and Box B, are 14.325 cm^3. Box A has a volume of 5.61 cm^3.

a. Let B represent the volume of Box B in cubic centimeters. Write an equation that could be used to determine the volume of Box B.

b. Solve the equation to determine the volume of Box B.

c. If the area of the base of Box B is 1.5 cm^2, write an equation that could be used to determine the height of Box B. Let h represent the height of Box B in centimeters.

d. Solve the equation to determine the height of Box B.

1. **A new company wants to mail out samples of its hair products. The company has a sample box that is a rectangular prism with a rectangular base with an area of $23\frac{1}{3}$ in². The height of the prism is $1\frac{1}{4}$ in. Determine the volume of the sample box.**

$$V = \text{Area of base} \times \text{height}$$
$$V = \left(23\frac{1}{3}\ \text{in}^2\right)\left(1\frac{1}{4}\ \text{in.}\right)$$
$$V = \frac{70}{3}\ \text{in}^2 \times \frac{5}{4}\ \text{in.}$$
$$V = \frac{350}{12}\ \text{in}^3$$

OR

$$V = \frac{175}{6}\ \text{in}^3$$

2. **A different sample box has a height that is twice as long as the original box described in Problem 1. What is the volume of this sample box? How does the volume of this sample box compare to the volume of the sample box in Problem 1?**

$$\text{V} = \text{Area of base} \times \text{height}$$
$$V = \left(23\frac{1}{3}\ \text{in}^2\right)\left(2\frac{1}{2}\ \text{in.}\right)$$
$$V = \left(\frac{70}{3}\ \text{in}^2\right)\left(\frac{5}{2}\ \text{in.}\right)$$
$$V = \frac{350}{6}\ \text{in}^3$$

OR

$$V = \frac{175}{3}\ \text{in}^3$$

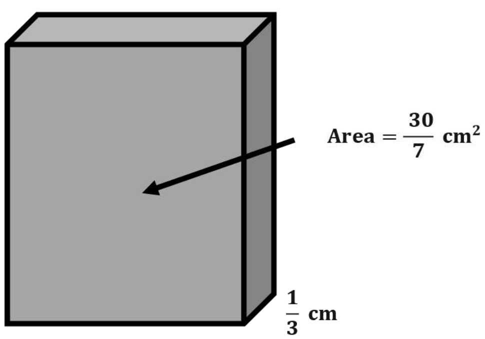
By doubling the height, we have also doubled the volume.

1. **Determine the volume of the rectangular prism.**

$$V = \text{Area of base} \times \text{height}$$
$$V = \left(\frac{30}{7}\ \text{cm}^2\right)\left(\frac{1}{3}\text{cm}\right)$$
$$V = \frac{30}{21}\ \text{cm}^3$$

OR

$$V = \frac{10}{7}\text{cm}^3$$



2. **Determine the volume of the rectangular prism in Problem 1 if the height is quadrupled (multiplied by four). Then, determine the relationship between the volumes in Problem 1 and this prism.**

$$V = \text{Area of base} \times \text{height}$$
$$V = \left(\frac{30}{7}\ \text{cm}^2\right)\left(\frac{4}{3}\text{cm}\right)$$
$$V = \frac{120}{21}\ \text{cm}^3$$

OR

$$V = \frac{40}{7}\ \text{cm}^3$$

When the height was quadrupled, the volume was also quadrupled.

3. **The area of the base of a rectangular prism can be represented by B, and the height is represented by h.**

   a. **Write an equation that represents the volume of the prism.**

   $V = Bh$

   b. **If the area of the base is doubled, write an equation that represents the volume of the prism.**

   $V = 2Bh$

   c. **If the height of the prism is doubled, write an equation that represents the volume of the prism.**

   $V = B2h = 2Bh$

   d. **Compare the volume in parts (b) and (c). What do you notice about the volumes?**

   The expressions in part (b) and part (c) are equal to each other.

   e. **Write an expression for the volume of the prism if both the height and the area of the base are doubled.**

   $V = 2B2h = 4Bh$

4. **Determine the volume of a cube with a side length of $5\frac{1}{3}$ in.**

$$V = l\,w\,h$$
$$V = \left(5\frac{1}{3}\ \text{in.}\right)\left(5\frac{1}{3}\ \text{in.}\right)\left(5\frac{1}{3}\ \text{in.}\right)$$
$$V = \frac{16}{3}\ \text{in.} \times \frac{16}{3}\ \text{in.} \times \frac{16}{3}\ \text{in.}$$
$$V = \frac{4096}{27}\ \text{in}^3$$

5. **Use the information in Problem 4 to answer the following:**

   a. **Determine the volume of the cube in Problem 4 if all of the side lengths are cut in half.**

$$V = l\,w\,h$$
$$V = \left(2\frac{2}{3}\ \text{in.}\right)\left(2\frac{2}{3}\ \text{in.}\right)\left(2\frac{2}{3}\ \text{in.}\right)$$
$$V = \frac{8}{3}\ \text{in.} \times \frac{8}{3}\ \text{in.} \times \frac{8}{3}\ \text{in.}$$
$$V = \frac{512}{27}\ \text{in}^3$$

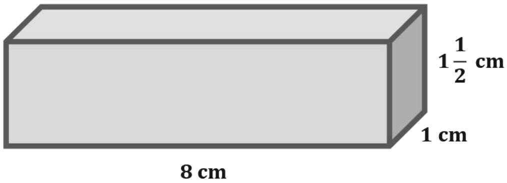
b. **How could you determine the volume of the cube with the side lengths cut in half using the volume in Problem 4?**

Because each side is half as long, I know that the volume will be $\frac{1}{8}$ the volume of the cube in Problem 4. This is because the length, the width, and the height were all cut in half.

$$\frac{1}{2}\mathrm{l}\times\frac{1}{2}\mathrm{w}\times\frac{1}{2}\mathrm{h}=\frac{1}{8}\mathrm{lwh}$$

$$\frac{1}{8}\times\frac{4096}{27}\ \mathrm{in}^3=\frac{512}{27}\ \mathrm{in}^3$$

6. **Use the rectangular prism to answer the following questions.**



a. **Complete the table.**

Length	Volume
$l=8$ cm	$12\ \mathrm{cm}^3$
$\frac{1}{2}l=4$ cm	$6\ \mathrm{cm}^3$
$\frac{1}{3}l=\frac{8}{3}$ cm	$4\ \mathrm{cm}^3$
$\frac{1}{4}l=2$ cm	$3\ \mathrm{cm}^3$
$2l=16$ cm	$24\ \mathrm{cm}^3$
$3l=24$ cm	$36\ \mathrm{cm}^3$
$4l=32$ cm	$48\ \mathrm{cm}^3$

b. **How did the volume change when the length was one-third as long?**

4 *is one-third of* 12. *Therefore, when the length is one-third as long, the volume is also one-third as much.*

c. **How did the volume change when the length was tripled?**

36 *is three times as much as* 12. *Therefore, when the length is three times as long, the volume is also three times as much.*

d. **What conclusion can you make about the relationship between the volume and the length?**

When the length changes but the width and height stay the same, the change in the volume is proportional to the change in the length.

7. The sum of the volumes of two rectangular prisms, Box A and Box B, are 14.325 cm^3. Box A has a volume of 5.61 cm^3.

a. Let B represent the volume of Box B in cubic centimeters. Write an equation that could be used to determine the volume of Box B.

$14.325\text{ cm}^3 = 5.61\text{ cm}^3 + B$

b. Solve the equation to determine the volume of Box B.

$B = 8.715\text{ cm}^3$

c. If the area of the base of Box B is 1.5 cm^2, write an equation that could be used to determine the height of Box B. Let h represent the height of Box B in centimeters.

$8.715\text{ cm}^3 = (1.5\text{ cm}^2)h$

d. Solve the equation to determine the height of Box B.

$h = 5.81\text{ cm}$