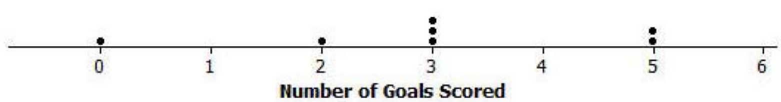
Name ____ Date__________________

The Mean as a Balance Point

1. If a class of 27 students has a mean score of 72 on a test, what is the sum of the 27 deviations of the scores from 72?

2. The dot plot below shows the number of goals scored by a school's soccer team in 7 games so far this season.



Use the "balancing" process to explain why the mean number of goals scored is 3. List all of the deviations and calculate the sum of the deviations. Explain your answer.

1. The number of pockets in the clothes worn by four students to school today is $4, 1, 3, 4$.

   a. Perform the “fair share” process to find the mean number of pockets for these four students. Sketch the cube representations for each step of the process.

   b. Find the sum of the deviations to prove the mean found in part (a) is correct.

2. The times (rounded to the nearest minute) it took each of six classmates to run a mile are $7, 9, 10, 11, 11,$ and 12 minutes.

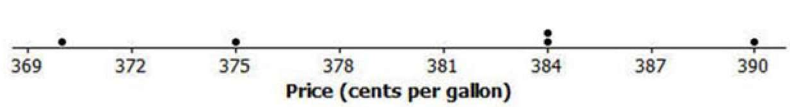
   a. Draw a dot plot representation for the times. Suppose that Sabina thinks the mean is 11 minutes. Use the sum of the deviations to show Sabina that the balance point of 11 is too high.

   b. Sabina now thinks the mean is 9 minutes. Use the sum of the deviations to verify that 9 is too small to be the mean number of pockets.

   c. Sabina asks you to find the mean by using the balancing process. Demonstrate that the mean is 10 minutes.

3. The prices per gallon of gasoline (in cents) at five stations across town on one day are shown in the following dot plot. The price for a sixth station is missing, but the mean price for all six stations was reported to be 380 cents per gallon. Use the “balancing” process to determine the price of a gallon of gasoline at the sixth station?

Dot Plot of Price (cents per gallon)



4. The number of phones (landline and cell) owned by the members of each of nine families is $3, 5, 5, 5, 6, 6, 6, 6, 8$.

   a. Use the mathematical formula for the mean (sum the data points and divide by the number of data points) to find the mean number of phones owned for these nine families.

   b. Draw a dot plot of the data and verify your answer in part (a) by using the "balancing" process and finding the sum of the deviations.

1. If a class of 27 students has a mean score of 72 on a test, what is the sum of the 27 deviations of the scores from 72?

   The sum is 0.

2. The dot plot below shows the number of goals that a school's soccer team has scored in 7 games so far this season.

   Use the "balancing" process to explain why the mean number of goals scored is 3. List all of the deviations and calculate the sum of the deviations. Explain your answer.

   The deviation from 0 to 3 is -3; from 2 to 3 is -1; from 5 to 3 is $+2$, for each of the two data points. The sum of the deviations is 0, since $-3 + (-1) + 2(+2) = 0$. The mean is 3.

1. The number of pockets in the clothes worn by four students to school today is $4, 1, 3, 4$.

   a. Perform the "fair share" process to find the mean number of pockets for these four students. Sketch the cube representations for each step of the process.

      Each of the 4's gives up a pocket to the person with one pocket, yielding three common pockets. The mean is 3 pockets.

   b. Find the sum of the deviations to prove the mean found in part (a) is correct.

      The 1-pocket data point has a deviation of -2. Each of the two 4-pocket data points has a deviation of $+1$. So, the sum of deviations is 0.

2. The times (rounded to the nearest minute) it took each of six classmates to run a mile are $7, 9, 10, 11, 11$, and 12 minutes.

   a. Draw a dot plot representation for the times. Suppose that Sabina thinks the mean is 11 minutes. Use the sum of the deviations to show Sabina that the balance point of 11 is too high.

      7 has a deviation of -4 from 11; 9 has a deviation of -2; 10 has a deviation of -1; each of the 11's has a deviation of 0; 12 has a deviation of $+1$. The sum of the deviations is -6. That indicates that 11 is too high.

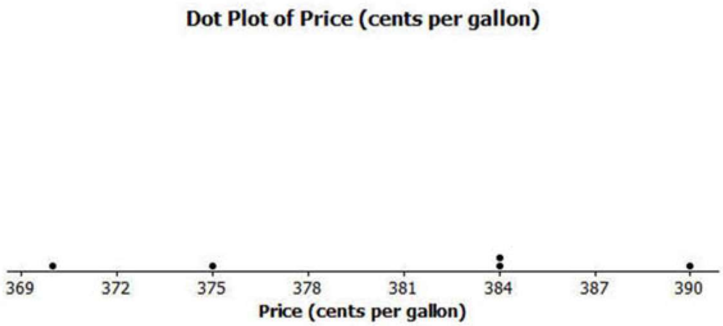
   b. Sabina now thinks the mean is 9 minutes. Use the sum of the deviations to verify that 9 is too small to be the mean number of minutes.

      7 has a deviation of -2 from 9; 9 has a deviation of 0; 10 has a deviation of $+1$; each of the 11's has a deviation of $+2$; 13 has a deviation of $+4$. The sum of the deviations is $+7$; therefore, 9 is too low for the mean.

   c. Sabina asks you to find the mean by using the balancing process. Demonstrate that the mean is 10 minutes.

      As 9 is too low, and 11 too high, try 10. The sum of the deviations is 0. So, the mean is 10 minutes.

3. **The prices per gallon of gasoline (in cents) at five stations across town on one day are shown in the following dot plot. The price for a sixth station is missing, but the mean price for all six stations was reported to be 380 cents per gallon. Use the "balancing" process to determine the price of a gallon of gasoline at the sixth station?**



The sum of the negative deviations from 380 is $(370-380)+(375-380)=-15$ cents. The sum of the positive deviations from 380 is $2(384-380)+(390-380)=+18$. So, the sixth station has to have a deviation that will cause the sum of the negative deviations plus the sum of the positive deviations to be 0. The deviation from 380 for the sixth station has to be -3. Therefore, the price of gasoline at the sixth station must be 377 cents.

Note: Try to keep your students from using the mathematical formula for the mean to solve this problem. They could, however, use it to check the answer they get from the balancing process.

4. **The number of phones (landline and cell) owned by the members of each of nine families is $3, 5, 5, 5, 6, 6, 6, 6, 8$.**

   a. **Use the mathematical formula for the mean (sum the data points and divide by the number of data points) to find the mean number of phones owned for these nine families.**

   The mean is $\frac{50}{9}=5\frac{5}{9}$ phones.

   b. **Draw a dot plot of the data and verify your answer in part (a) by using the "balancing" process and finding the sum of the deviations.**

   The sum of the negative deviations from $5\frac{5}{9}$ is: $\left(3-5\frac{5}{9}\right)+3\left(5-5\frac{5}{9}\right)=-4\frac{2}{9}$.

   The sum of the positive deviations from $5\frac{5}{9}$ is: $4\left(6-5\frac{5}{9}\right)+\left(8-5\frac{5}{9}\right)=-4\frac{2}{9}$.

   The sum of the deviations is 0, so the mean is $5\frac{5}{9}$ phones.