Name _____ Date_____

# The Mean Absolute Deviation (MAD)

1. The Mean Absolute Deviation (MAD) is a measure of variability for a data set. What does a data distribution look like if its MAD equals zero? Explain.

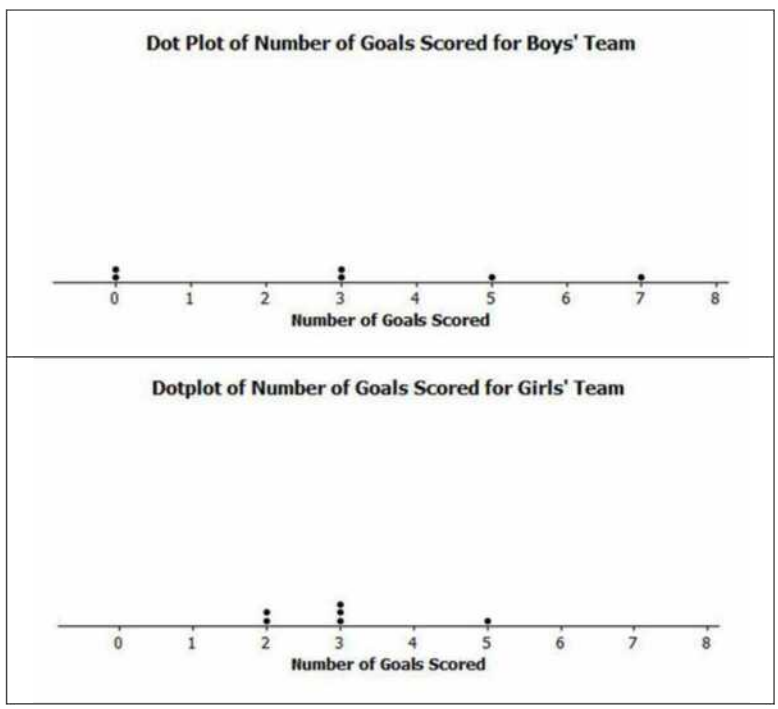
2. Is it possible to have a negative value for the MAD of a data set?

3. Suppose that seven students have the following number of pets: 1, 1, 1, 2, 4, 4, 8.
   a. The mean number of pets for these seven students is three pets. Use the following table to find the MAD number of pets for this distribution of number of pets.

| Student | # of Pets | Deviations | Absolute Deviations |
|---|---|---|---|
| 1 | 1 | | |
| 2 | 1 | | |
| 3 | 1 | | |
| 4 | 2 | | |
| 5 | 4 | | |
| 6 | 4 | | |
| 7 | 8 | | |
| Sum | | | |

   b. Explain in words what the MAD means for this data set.

1. Suppose the dot plot on the left shows the number of goals a boys' soccer team has scored in six games so far this season, and the dot plot on the right shows the number of goals a girls' soccer team has scored in six games so far this season. The mean for both of these teams is 3.




a. Before doing any calculations, which dot plot has the larger MAD? Explain how you know.

b. Use the following tables to find the MAD number of goals for each distribution. Round your calculations to the nearest hundredth.

| Boys' Team | | |
|---|---|---|
| #Goals | Deviations | Absolute Deviations |
| 0 | $-3$ | |
| 0 | $-3$ | |
| 3 | $3-3=0$ | |
| 3 | | |
| 5 | | |
| 7 | | |
| Sum | | |

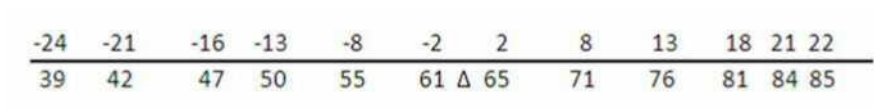
| Girls' Team | | |
|---|---|---|
| #Goals | Deviations | Absolute Deviations |
| 2 | | |
| 2 | | |
| 3 | | |
| 3 | | |
| 3 | | |
| 5 | | |
| Sum | | |

c. Based on the computed MAD values, for which distribution is the mean a better indication of a typical value? Explain your answer.

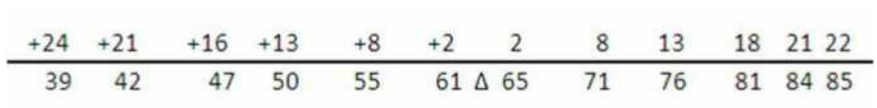
2. Recall Robert's problem of deciding whether to move to New York City or to San Francisco. The table of temperatures (in degrees Fahrenheit) and deviations for the New York City distribution is as follows:

| NYC | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp | 39 | 42 | 50 | 61 | 71 | 81 | 85 | 84 | 76 | 65 | 55 | 47 |
| Deviation | $-24$ | $-21$ | $-13$ | $-2$ | 8 | 18 | 22 | 21 | 13 | 2 | $-8$ | $-16$ |

a. The dot plot below is written with the deviations above each of the monthly temperatures. What is the sum of all of the deviations? Are you surprised? Explain.




b. The absolute deviations for the monthly temperatures are shown below. Use this information to calculate the MAD. Explain the MAD in words for this problem.




c. Complete the following table and then use the values to calculate the MAD for the San Francisco data distribution.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp | 57 | 60 | 62 | 63 | 64 | 67 | 67 | 68 | 70 | 69 | 63 | 58 |
| Deviations | | | | $-1$ | 0 | $+3$ | | | | | | |
| Absolute Deviations | | | | | | | | | | | | |

d. Comparing the MAD values for New York City and San Francisco, which city would Robert choose to move to if he is interested in having a lot of variability in monthly temperatures? Explain using the MAD.

3. Consider the following data of the number of green jellybeans in seven bags sampled from five different candy manufacturers (Awesome, Delight, Finest, Sweeties, YumYum). Note that the mean in each distribution is 42 green jellybeans.

| | Bag 1 | Bag 2 | Bag 3 | Bag 4 | Bag 5 | Bag 6 | Bag 7 |
|---|---|---|---|---|---|---|---|
| Awesome | 40 | 40 | 41 | 42 | 42 | 43 | 46 |
| Delight | 22 | 31 | 36 | 42 | 48 | 53 | 62 |
| Finest | 26 | 36 | 40 | 43 | 47 | 50 | 52 |
| Sweeties | 36 | 39 | 42 | 42 | 42 | 44 | 49 |
| YumYum | 33 | 36 | 42 | 42 | 45 | 48 | 48 |

a. Complete the following table of the deviations of the number of green jellybeans from the mean number of green jellybeans in the seven bags.

| | Bag 1 | Bag 2 | Bag 3 | Bag 4 | Bag 5 | Bag 6 | Bag 7 |
|---|---|---|---|---|---|---|---|
| Awesome | $-2$ | $-2$ | $-1$ | 0 | 0 | $+1$ | $+4$ |
| Delight | $-20$ | $-11$ | $-6$ | | | | |
| Finest | $-16$ | | | | | | |
| Sweeties | | | | | | | |
| YumYum | | | | | | | |

b. Based on what you learned about MAD, which manufacturer do you think will have the lowest MAD? Calculate the MAD for the manufacturer you selected.

1. The Mean Absolute Deviation (MAD) is a measure of variability for a data set. What does a data distribution look like if its MAD equals zero? Explain.

*If the MAD is zero, then all of the deviations are zero. For example, City A had a dot plot with all the same temperatures. They were all the same, so there was no variability, since the MAD measures average temperature from the mean. And it is zero, because all the deviations are zero.*

2. Is it possible to have a negative value for the MAD of a data set?

*Because a MAD is the average of the absolute values of the deviations, it is always a positive value.*

3. Suppose that seven students have the following number of pets: $1, 1, 1, 2, 4, 4, 8$.

   a. The mean number of pets for these seven students is three pets. Use the following table to find the MAD number of pets for this distribution of number of pets.

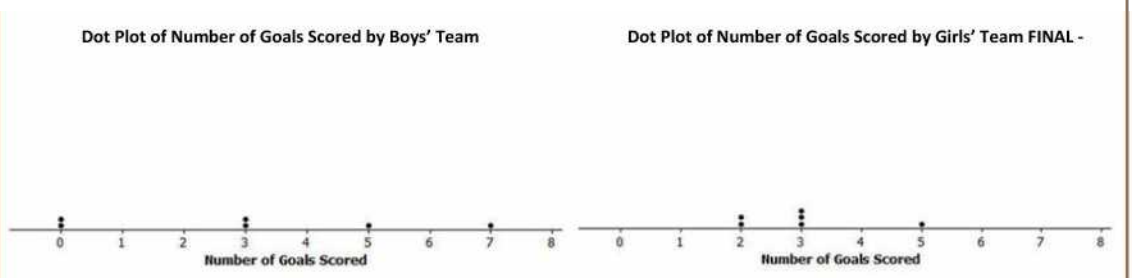
   *The MAD number of pets is* $+\frac{14}{7} = 2$ *pets.*

| Student | # of Pets | Deviations | Absolute Deviations |
|---|---|---|---|
| 1 | 1 | $1-3=-2$ | $+2$ |
| 2 | 1 | $1-3=-2$ | $+2$ |
| 3 | 1 | $1-3=-2$ | $+2$ |
| 4 | 2 | $2-3=-1$ | $+1$ |
| 5 | 4 | $4-3=+1$ | $+1$ |
| 6 | 4 | $4-3=+1$ | $+1$ |
| 7 | 8 | $8-3=+5$ | $+5$ |
| Sum | | 0 | $+14$ |

   b. Explain in words what the MAD means for this data set.

   *On average, these students' number of pets differ by 2 pets from their mean of 3 pets.*

1. Suppose the dot plot on the left shows the number of goals a boys' soccer team has scored in six games so far this season, and the dot plot on the right shows the number of goals a girls' soccer team has scored in six games so far this season. The mean for both of these teams is 3.




   a. Before doing any calculations, which dot plot has the larger MAD? Explain how you know.

   *The graph of the Boys' team is more spread out and has the larger deviations from the mean.*

b. Use the following tables to find the MAD number of goals for each distribution. Round your calculations to the nearest hundredth.

| Boys' Team | | |
|---|---|---|
| #Goals | Deviations | Absolute Deviations |
| 0 | $-3$ | $+3$ |
| 0 | $-3$ | $+3$ |
| 3 | $3-3$ | 0 |
| 3 | $3-3=0$ | 0 |
| 5 | $5-3=2$ | $+2$ |
| 7 | $7-3=4$ | $+4$ |
| Sum | | $+12$ |

| Girls' Team | | |
|---|---|---|
| #Goals | Deviations | Absolute Deviations |
| 2 | $2-3=-1$ | $+1$ |
| 2 | $2-3=-1$ | $+1$ |
| 3 | $3-3=0$ | 0 |
| 3 | $3-3=0$ | 0 |
| 3 | $3-3=0$ | 0 |
| 5 | $5-3=2$ | $+2$ |
| Sum | | $+4$ |

c. Based on the computed MAD values, for which distribution is the mean a better indication of a typical value? Explain your answer.

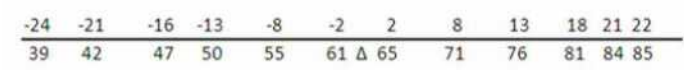
*The Girls' team, because the measure of variability given by the MAD is lower ($0.67$ goals) than the Boys' MAD ($2$ goals). Visually, the data in the Girls' dot plot are more compact around the mean than they are in the Boys' dot plot.*

2. Recall Robert's problem of deciding whether to move to New York City or to San Francisco. The table of temperatures (in degrees Fahrenheit) and deviations for the New York City distribution is as follows:

| NYC | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp | 39 | 42 | 50 | 61 | 71 | 81 | 85 | 84 | 76 | 65 | 55 | 47 |
| Deviation | $-24$ | $-21$ | $-13$ | $-2$ | 8 | 18 | 22 | 21 | 13 | 2 | $-8$ | $-16$ |

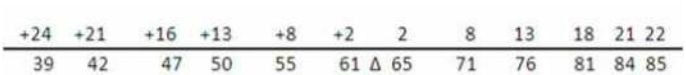
a. The dot plot below is written with the deviations above each of the monthly temperatures. What is the sum of all of the deviations? Are you surprised? Explain.

*The sum of the negative deviations is: $-24+-21+-16+-13+-8+-2=-84$. The sum of the positive deviations is: $2+8+13+18+21+22=+84$. The sum of all of the deviations is $-84+84=0$. Students should not be surprised, since the sum of the deviations of any data set around its mean is $0$.*




b. The absolute deviations for the monthly temperatures are shown below. Use this information to calculate the MAD. Explain the MAD in words for this problem.

*The sum of the absolute deviations is $2(84)=168$ degrees. The average of the absolute deviations, MAD, is $\frac{168}{12}=14$ degrees. On average, the monthly temperatures in New York City differ from the mean of $63$ degrees by $14$ degrees.*



c. Complete the following table and then use the values to calculate the MAD for the San Francisco data distribution.

*First of all, note that the sum of the negative deviations is $-21$, and the sum of the positive deviations is $+21$, as it should be. The sum of the absolute deviations is $2(21) = 42$. The MAD is the mean of the absolute deviations, which equals $\frac{42}{12} = 3.5$ degrees.*

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp | 57 | 60 | 62 | 63 | 64 | 67 | 67 | 68 | 70 | 69 | 63 | 58 |
| Deviations | $-7$ | $-4$ | $-2$ | $-1$ | 0 | $+3$ | $+3$ | $+4$ | $+6$ | $+5$ | $-1$ | $-6$ |
| Absolute Deviations | $+7$ | $+4$ | $+2$ | $+1$ | 0 | $+3$ | $+3$ | $+4$ | $+6$ | $+5$ | $+1$ | $+6$ |

d. Comparing the MAD values for New York City and San Francisco, which city would Robert choose to move to if he is interested in having a lot of variability in monthly temperatures? Explain using the MAD.

*New York City has a MAD of 14 degrees, as compared to 3.5 degrees in San Francisco. Robert should choose New York City if he wants to have more variability in monthly temperatures.*

3. Consider the following data of the number of green jellybeans in seven bags sampled from five different candy manufacturers (Awesome, Delight, Finest, Sweeties, YumYum). Note that the mean in each distribution is 42 green jellybeans.

| | Bag 1 | Bag 2 | Bag 3 | Bag 4 | Bag 5 | Bag 6 | Bag 7 |
|---|---|---|---|---|---|---|---|
| Awesome | 40 | 40 | 41 | 42 | 42 | 43 | 46 |
| Delight | 22 | 31 | 36 | 42 | 48 | 53 | 62 |
| Finest | 26 | 36 | 40 | 43 | 47 | 50 | 52 |
| Sweeties | 36 | 39 | 42 | 42 | 42 | 44 | 49 |
| YumYum | 33 | 36 | 42 | 42 | 45 | 48 | 48 |

a. Complete the following table of the deviations of the number of green jellybeans from the mean number of green jellybeans in the seven bags.

| | Bag 1 | Bag 2 | Bag 3 | Bag 4 | Bag 5 | Bag 6 | Bag 7 |
|---|---|---|---|---|---|---|---|
| Awesome | $-2$ | $-2$ | $-1$ | 0 | 0 | $+1$ | $+4$ |
| Delight | $-20$ | $-11$ | $-6$ | 0 | $+6$ | $+11$ | $+20$ |
| Finest | $-16$ | $-6$ | $-2$ | $+1$ | $+5$ | $+8$ | $+10$ |
| Sweeties | $-6$ | $-3$ | 0 | 0 | 0 | $+2$ | $+7$ |
| YumYum | $-9$ | $-6$ | 0 | 0 | $+3$ | $+6$ | $+6$ |

b. Based on what you learned about MAD, which manufacturer do you think will have the lowest MAD? Calculate the MAD for the manufacturer you selected.

*Use the MAD for each manufacturer to evaluate students' responses.*

| | *Bag 1* | *Bag 2* | *Bag 3* | *Bag 4* | *Bag 5* | *Bag 6* | *Bag 7* | *SUM* | *MAD* |
|---|---|---|---|---|---|---|---|---|---|
| *Awesome* | $+2$ | $+2$ | $+1$ | 0 | 0 | $+1$ | $+4$ | $+10$ | 1.4 |
| *Delight* | $+20$ | $+11$ | $+6$ | 0 | $+6$ | $+11$ | $+20$ | $+74$ | 10.6 |
| *Finest* | $+16$ | $+6$ | $+2$ | $+1$ | $+5$ | $+8$ | $+10$ | $+48$ | 6.9 |
| *Sweeties* | $+6$ | $+3$ | 0 | 0 | 0 | $+2$ | $+7$ | $+18$ | 2.6 |
| *YumYum* | $+9$ | $+6$ | 0 | 0 | $+3$ | $+6$ | $+6$ | $+30$ | 4.3 |