

Name _____

Date _____

Solving Equations Using Algebra

Andrew's math teacher entered the seventh-grade students in a math competition. There was an enrollment fee of \$30 and also an \$11 charge for each packet of 10 tests. The total cost was \$151. How many tests were purchased?

Set up an equation to model this situation, solve it using if-then statements, and justify the reasons for each step in your solution.

For Exercises 1–4, solve each equation algebraically using if-then statements to justify your steps.

1. $\frac{2}{3}x - 4 = 20$

2. $4 = \frac{-1+x}{2}$

3. $12(x + 9) = -108$

4. $5x + 14 = -7$

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

5. A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If $\frac{3}{4}$ of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?
6. Bob's monthly phone bill is made up of a \$10 fee plus \$0.05 per minute. Bob's phone bill for July was \$22. Write an equation to model the situation, using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.
7. Kym switched cell phone plans. She signed up for a new plan that will save her \$3.50 per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is \$294. How much did Kym pay per month under her old phone plan?

Andrew's math teacher entered the seventh-grade students in a math competition. There was an enrollment fee of \$30 and also an \$11 charge for each packet of 10 tests. The total cost was \$151. How many tests were purchased?

Set up an equation to model this situation, solve it using if-then statements, and justify the reasons for each step in your solution.

Let p = the number of test packets.

Enrollment fee + cost of test = 151

If: $30 + 11p = 151$

Then: $30 - 30 + 11p = 151 - 30$ Subtraction property of equality for the additive inverse of 30

If: $0 + 11p = 121$

Then: $11p = 121$ Additive identity

If: $11p = 121$

Then: $\frac{1}{11}(11p) = \frac{1}{11}(121)$ Multiplication property of equality using the multiplicative inverse of 11

If: $1p = 11$

Then: $p = 11$ Multiplicative identity

Andrew's math teacher bought 11 packets of tests. There were 10 tests in each packet, and $10 \times 11 = 110$.

So, there were 110 tests purchased.

For Exercises 1–4, solve each equation algebraically using if-then statements to justify your steps.

1. $\frac{2}{3}x - 4 = 20$

If: $\frac{2}{3}x - 4 = 20$

Then: $\frac{2}{3}x - 4 + 4 = 20 + 4$ Addition property of equality using the additive inverse of -4

If: $\frac{2}{3}x + 0 = 24$

Then: $\frac{2}{3}x = 24$ Additive identity

If: $\frac{2}{3}x = 24$

Then: $\left(\frac{3}{2}\right)\frac{2}{3}x = \left(\frac{3}{2}\right)24$ Multiplication property of equality using the multiplicative inverse of $\frac{2}{3}$

If: $1x = 36$

Then: $x = 36$ Multiplicative identity

2. $4 = \frac{-1+x}{2}$

If: $4 = \frac{-1+x}{2}$

Then: $2(4) = 2\left(\frac{-1+x}{2}\right)$ *Multiplication property of equality using the multiplicative inverse of $\frac{1}{2}$*

If: $8 = 1(-1+x)$

Then: $8 = -1+x$ *Multiplicative identity*

If: $8 = -1+x$

Then: $8 - (-1) = -1 - (-1) + x$ *Subtraction property of equality for the additive inverse of -1*

If: $9 = 0+x$

Then: $9 = x$ *Additive identity*

3. $12(x+9) = -108$

If: $12(x+9) = -108$

Then: $\left(\frac{1}{12}\right)12(x+9) = \left(\frac{1}{12}\right)(-108)$ *Multiplication property of equality using the multiplicative inverse of 12*

If: $1(x+9) = -9$

Then: $x+9 = -9$ *Multiplicative identity*

If: $x+9 = -9$

Then: $x+9-9 = -9-9$ *Subtraction property of equality for the additive inverse of 9*

If: $x+0 = -18$

Then: $x = -18$ *Additive identity*

4. $5x + 14 = -7$

If: $5x + 14 = -7$

Then: $5x + 14 - 14 = -7 - 14$ *Subtraction property of equality for the additive inverse of 14*

If: $5x + 0 = -21$

Then: $5x = -21$ *Additive identity*

If: $5x = -21$

Then: $\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)(-21)$ *Multiplication property of equality using the multiplicative inverse of 5*

If: $1x = -4.2$

Then: $x = -4.2$ *Multiplicative identity*

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

5. A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If $\frac{3}{4}$ of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?

Let x = the longer piece of pipe

$$\text{If: } \frac{3}{4}(x - 12) = 120$$

$$\text{Then: } \frac{4}{3}\left(\frac{3}{4}\right)(x - 12) = \left(\frac{4}{3}\right)120 \quad \text{Multiplication property of equality using the multiplicative inverse of } \frac{3}{4}$$

$$\text{If: } 1(x - 12) = 160$$

$$\text{Then: } x - 12 = 160 \quad \text{Multiplicative identity}$$

$$\text{If: } x - 12 = 160$$

$$\text{Then: } x - 12 + 12 = 160 + 12 \quad \text{Addition property of equality for the additive inverse of } -12$$

$$\text{If: } x + 0 = 172$$

$$\text{Then: } x = 172 \quad \text{Additive identity}$$

The longer piece of pipe is 172 ft.

6. Bob's monthly phone bill is made up of a \$10 fee plus \$0.05 per minute. Bob's phone bill for July was \$22. Write an equation to model the situation using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.

Let m = the number of phone minutes Bob used

$$\text{If: } 10 + 0.05m = 22$$

$$\text{Then: } 10 - 10 + 0.05m = 22 - 10 \quad \text{Subtraction property of equality for the additive inverse of 10}$$

$$\text{If: } 0 + 0.05m = 12$$

$$\text{Then: } 0.05m = 12 \quad \text{Additive identity}$$

$$\text{If: } 0.05m = 12$$

$$\text{Then: } \left(\frac{1}{0.05}\right)0.05m = \left(\frac{1}{0.05}\right)12 \quad \text{Multiplication property of equality using the multiplicative inverse of 0.05}$$

$$\text{If: } 1m = 240$$

$$\text{Then: } m = 240 \quad \text{Multiplicative identity}$$

Bob used 240 phone minutes in July.

7. Kym switched cell phone plans. She signed up for a new plan that will save her \$3.50 per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is \$294. How much did Kym pay per month under her old phone plan?

Let n = the amount Kym paid per month for her old cell phone plan

If: $294 = 12(n - 3.50)$

Then: $\left(\frac{1}{12}\right)(294) = \left(\frac{1}{12}\right)12(n - 3.50)$ *Multiplication property of equality using the multiplicative inverse of 12*

If: $24.5 = 1(n - 3.50)$

Then: $24.5 = n - 3.50$ *Multiplicative identity*

If: $24.5 = n - 3.50$

Then: $24.5 + 3.50 = n - 3.50 + 3.50$ *Addition property of equality for the additive inverse of -3.50*

If: $28 = n + 0$

Then: $28 = n$ *additive identity*

Kym paid \$28 per month for her old cell phone plan.