

Name \_\_\_\_\_

Date \_\_\_\_\_

## Inequalities

Shaggy earned \$7.55 per hour plus an additional \$100 in tips waiting tables on Saturday. He earned at least \$160 in all. Write an inequality and find the minimum number of hours, to the nearest hour, that Shaggy worked on Saturday.

1. Match each problem to the inequality that models it. One choice will be used twice.

- |       |   |                      |
|-------|---|----------------------|
| _____ | The sum of three times a number and $-4$ is greater than $17$ . | a. $3x + -4 \geq 17$ |
| _____ | The sum of three times a number and $-4$ is less than $17$ .    | b. $3x + -4 < 17$    |
| _____ | The sum of three times a number and $-4$ is at most $17$ .      | c. $3x + -4 > 17$    |
| _____ | The sum of three times a number and $-4$ is no more than $17$ . | d. $3x + -4 \leq 17$ |
| _____ | The sum of three times a number and $-4$ is at least $17$ .     |                      |

2. If  $x$  represents a positive integer, find the solutions to the following inequalities.

- |                    |                          |
|--------------------|--------------------------|
| a. $x < 7$         | f. $-x \geq 2$           |
| b. $x - 15 < 20$   | g. $\frac{x}{3} < 2$     |
| c. $x + 3 \leq 15$ | h. $-\frac{x}{3} > 2$    |
| d. $-x > 2$        | i. $3 - \frac{x}{4} > 2$ |
| e. $10 - x > 2$    |                          |

3. Recall that the symbol  $\neq$  means "not equal to." If  $x$  represents a positive integer, state whether each of the following statements is always true, sometimes true, or false.

- |             |                |
|-------------|----------------|
| a. $x > 0$  | e. $x \geq 1$  |
| b. $x < 0$  | f. $x \neq 0$  |
| c. $x > -5$ | g. $x \neq -1$ |
| d. $x > 1$  | h. $x \neq 5$  |

4. Twice the smaller of two consecutive integers increased by the larger integer is at least  $25$ .

Model the problem with an inequality, and determine which of the given values  $7$ ,  $8$ , and/or  $9$  are solutions. Then find the smallest number that will make the inequality true.

5.

- The length of a rectangular fenced enclosure is  $12$  feet more than the width. If Farmer Dan has  $100$  feet of fencing, write an inequality to find the dimensions of the rectangle with the largest perimeter that can be created using  $100$  feet of fencing.
- What are the dimensions of the rectangle with the largest perimeter? What is the area enclosed by this rectangle

6. At most, Kyle can spend  $\$50$  on sandwiches and chips for a picnic. He already bought chips for  $\$6$  and will buy sandwiches that cost  $\$4.50$  each. Write and solve an inequality to show how many sandwiches he can buy. Show your work and interpret your solution.

Shaggy earned \$7.55 per hour plus an additional \$100 in tips waiting tables on Saturday. He earned at least \$160 in all. Write an inequality and find the minimum number of hours, to the nearest hour, that Shaggy worked on Saturday.

Let  $h$  represent the number of hours worked.

$$\begin{aligned}
 7.55h + 100 &\geq 160 \\
 7.55h + 100 - 100 &\geq 160 - 100 \\
 7.55h &\geq 60 \\
 \left(\frac{1}{7.55}\right)(7.55h) &\geq \left(\frac{1}{7.55}\right)(60) \\
 h &\geq 7.9
 \end{aligned}$$

If Shaggy earned at least \$160, he would have worked at least 8 hours.

Note: The solution shown above is rounded to the nearest tenth. The overall solution, though, is rounded to the nearest hour since that is what the question asks for.

1. Match each problem to the inequality that models it. One choice will be used twice.

- |  |                      |
|--|----------------------|
| <u>      </u> $c$ The sum of three times a number and $-4$ is greater than 17. | a. $3x + -4 \geq 17$ |
| <u>      </u> $b$ The sum of three times a number and $-4$ is less than 17.    | b. $3x + -4 < 17$    |
| <u>      </u> $d$ The sum of three times a number and $-4$ is at most 17.      | c. $3x + -4 > 17$    |
| <u>      </u> $d$ The sum of three times a number and $-4$ is no more than 17. | d. $3x + -4 \leq 17$ |
| <u>      </u> $a$ The sum of three times a number and $-4$ is at least 17.     |                      |

2. If  $x$  represents a positive integer, find the solutions to the following inequalities.

- |                                     |                                   |
|-------------------------------------|-----------------------------------|
| a. $x < 7$<br>$x < 7$               | b. $x - 15 < 20$<br>$x < 35$      |
| c. $x + 3 \leq 15$<br>$x \leq 12$   | d. $-x > 2$<br>$x < -2$           |
| e. $10 - x > 2$<br>$x < 8$          | f. $-x \geq 2$<br>$x \leq -2$     |
| g. $\frac{x}{3} < 2$<br>$x < 6$     | h. $-\frac{x}{3} > 2$<br>$x < -6$ |
| i. $3 - \frac{x}{4} > 2$<br>$x < 4$ |                                   |

3. Recall that the symbol  $\neq$  means "not equal to." If  $x$  represents a positive integer, state whether each of the following statements is always true, sometimes true, or false.

a.  $x > 0$

*Always true*

b.  $x < 0$

*False*

c.  $x > -5$

*Always true*

d.  $x > 1$

*Sometimes true*

e.  $x \geq 1$

*Always true*

f.  $x \neq 0$

*Always true*

g.  $x \neq -1$

*Always true*

h.  $x \neq 5$

*Sometimes true*

4. Twice the smaller of two consecutive integers increased by the larger integer is at least 25.

Model the problem with an inequality, and determine which of the given values 7, 8, and/or 9 are solutions. Then, find the smallest number that will make the inequality true.

$$2x + x + 1 \geq 25$$

$$x = 7$$

$$2x + x + 1 \geq 25$$

$$2(7) + 7 + 1 \geq 25$$

$$14 + 7 + 1 \geq 25$$

$$22 \geq 25$$

*False*

$$x = 8$$

$$2x + x + 1 \geq 25$$

$$2(8) + 8 + 1 \geq 25$$

$$16 + 8 + 1 \geq 25$$

$$25 \geq 25$$

*True*

$$x = 9$$

$$2x + x + 1 \geq 25$$

$$2(9) + 9 + 1 \geq 25$$

$$18 + 9 + 1 \geq 25$$

$$28 \geq 25$$

*True*

*The smallest integer would be 8.*

5.

a. The length of a rectangular fenced enclosure is 12 feet more than the width. If Farmer Dan has 100 feet of fencing, write an inequality to find the dimensions of the rectangle with the largest perimeter that can be created using 100 feet of fencing.

*Let  $w$  represent the width of the fenced enclosure.*

*$w + 12$ : length of the fenced enclosure*

$$w + w + w + 12 + w + 12 \leq 100$$

$$4w + 24 \leq 100$$

- b. What are the dimensions of the rectangle with the largest perimeter? What is the area enclosed by this rectangle?

$$\begin{aligned}4w + 24 &\leq 100 \\4w + 24 - 24 &\leq 100 - 24 \\4w + 0 &\leq 76 \\ \left(\frac{1}{4}\right)(4w) &\leq \left(\frac{1}{4}\right)(76) \\w &\leq 19\end{aligned}$$

*maximum width is 19 feet*

*maximum length is 31 feet*

*maximum area:*      $A = lw$   
 $A = (19)(31)$   
 $A = 589 \text{ sq. ft.}$

6. At most, Kyle can spend \$50 on sandwiches and chips for a picnic. He already bought chips for \$6 and will buy sandwiches that cost \$4.50 each. Write and solve an inequality to show how many sandwiches he can buy. Show your work and interpret your solution.

*Let  $s$  represent the number of sandwiches.*

$$\begin{aligned}4.50s + 6 &\leq 50 \\4.50s + 6 - 6 &\leq 50 - 6 \\4.50s &\leq 44 \\ \left(\frac{1}{4.50}\right)(4.50s) &\leq \left(\frac{1}{4.50}\right)(44) \\s &\leq 9\frac{7}{9}\end{aligned}$$

*At most, Kyle can buy 9 sandwiches with \$50.*