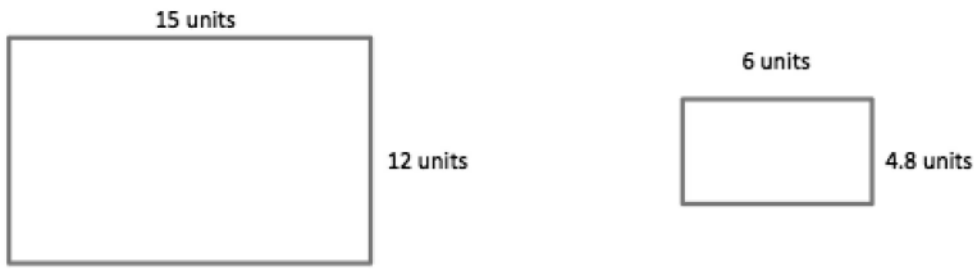
Name _____ Date _____

Solving Area Problems Using Scale Drawings

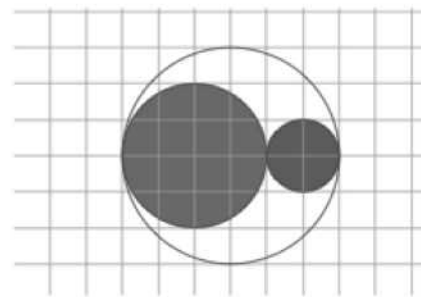
Write an equation relating the area of the original (larger) drawing to its smaller scale drawing. Explain how you determined the equation. What percent of the area of the larger drawing is the smaller scale drawing?



1. What percent of the area of the larger circle is shaded?

   a. Solve this problem using scale factors.

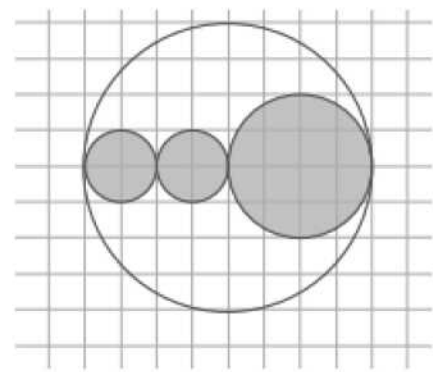
   b. Verify your work in part (a) by finding the actual areas.



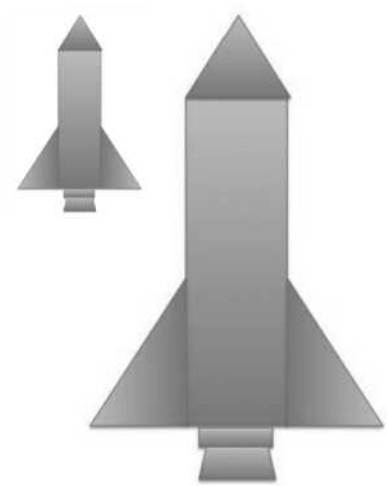
2. The area of the large disk is 50.24 units^2.

   a. Find the area of the shaded region using scale factors. Use 3.14 as an estimate for π.

   b. What percent of the large circular region is unshaded?

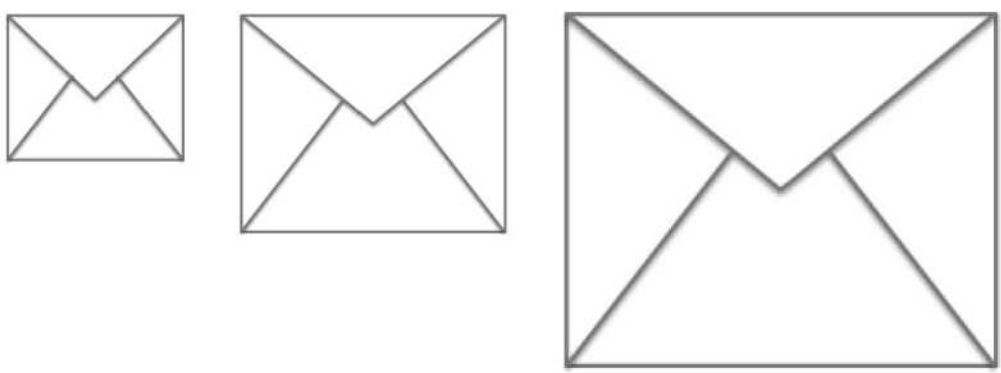


3. Ben cut the following rockets out of cardboard. The height from the base to the tip of the smaller rocket is 20 cm. The height from the base to the tip of the larger rocket is 120 cm. What percent of the area of the smaller rocket is the area of the larger rocket?



4. In the photo frame depicted below, three 5 inch by 5 inch squares are cut out for photographs. If these cut-out regions make up $\frac{3}{16}$ of the area of the entire photo frame, what are the dimensions of the photo frame?

5. Kelly was online shopping for envelopes for party invitations and saw these images on a website.



The website listed the dimensions of the small envelope as 6 in. by 8 in. and the medium envelope as 10 in. by $13\frac{1}{3}$ in.

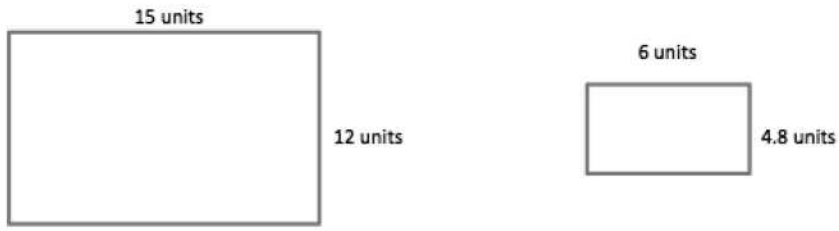
a. Compare the dimensions of the small and medium envelopes. If the medium envelope is a scale drawing of the small envelope, what is the scale factor?

b. If the large envelope was created based on the dimensions of the small envelope using a scale factor of 250%, find the dimensions of the large envelope.

c. If the medium envelope was created based on the dimensions of the large envelope, what scale factor was used to create the medium envelope?

d. What percent of the area of the larger envelope is the area of the medium envelope?

Write an equation relating the area of the original (larger) drawing to its smaller scale drawing. Explain how you determined the equation. What percent of the area of the larger drawing is the smaller scale drawing?



Scale factor:

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$\text{Scale Drawing Length} = \text{Percent} \times \text{Original Length}$$
$$6 = \text{Percent} \times 15$$
$$\frac{6}{15} = \frac{2}{5} = \frac{4}{10} = 0.4$$

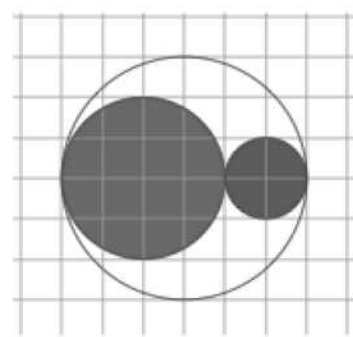
The area of the scale drawing is equal to the square of the scale factor times the area of the original drawing. Using A to represent the area of the original drawing, then the area of the scale is

$$\left(\frac{4}{10}\right)^2 A = \frac{16}{100}A.$$

As a percent, $\frac{16}{100}A = 0.16A.$

Therefore, the area of the scale drawing is 16% *of the area of the original drawing.*

1. **What percent of the area of the larger circle is shaded?**



   a. **Solve this problem using scale factors.**

   Scale factors:

   Shaded small circle: radius $= 1$ unit

   Shaded medium circle: radius $= 2$ units

   Large circle: radius $= 3$ units, *area* $= A$

   Area of small circle: $\left(\frac{1}{3}\right)^2 A = \frac{1}{9}A$

   Area of medium circle: $\left(\frac{2}{3}\right)^2 A = \frac{4}{9}A$

   Area of shaded region: $\frac{1}{9}A + \frac{4}{9}A = \frac{5}{9}A = \frac{5}{9} \times 100\%, A = 55\frac{5}{9}\%A$

   The area of the shaded region is $55\frac{5}{9}\%$ *of the area of the entire circle.*

b. Verify your work in part (a) by finding the actual areas.

Areas:

Small circle: $A = \pi r^2$

$A = \pi(1)^2 \text{ square units}$

$A = 1\pi \text{ square units}$

Medium circle: $A = \pi r^2$

$A = \pi(2)^2 \text{ square units}$

$A = 4\pi \text{ square units}$

Area of shaded circles: $1\pi + 4\pi = 5\pi$

Large circle: $A = \pi r^2$

$A = \pi(3)^2 \text{ square units}$

$A = 9\pi \text{ square units}$

Percent of shaded to large circle: $\frac{5\pi}{9\pi} = \frac{5}{9} = \frac{5}{9} \times 100\% = 55\frac{5}{9}\%$

2. The area of the large disk is 50.24 units^2.

a. Find the area of the shaded region using scale factors. Use 3.14 as an estimate for π.

Radius of small shaded circles $= 1 \text{ unit}$

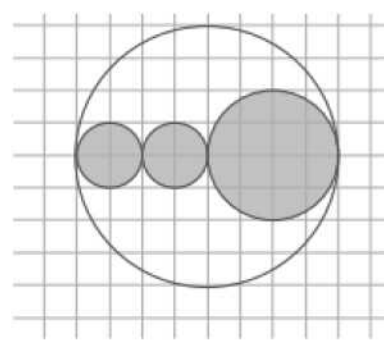
Radius of larger shaded circle $= 2 \text{ units}$

Radius of large disk $= 4 \text{ units}$

Scale factor of shaded region:

Small shaded circles: $\frac{1}{4}$

Large shaded circle: $\frac{2}{4}$



If A represents the area of the large disk, then the total shaded area:

$$\left(\frac{1}{4}\right)^2 A + \left(\frac{1}{4}\right)^2 A + \left(\frac{2}{4}\right)^2 A$$

$$\frac{1}{16}A + \frac{1}{16}A + \frac{4}{16}A = \frac{6}{16}A$$

$$\frac{6}{16}A = \frac{6}{16}(50.24) \text{ units}^2$$

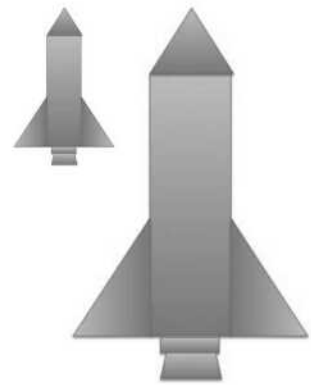
The area of the shaded region is 18.84 units^2.

b. What percent of the large circular region is unshaded?

Area of the shaded region is 18.84 square units. Area of total is 50.24 square units. Area of the unshaded region is 31.40 square units. Percent of large circular region that is unshaded is

$$\frac{31.4}{50.24} = \frac{5}{8} = 0.625 = 62.5\%$$

3. **Ben cut the following rockets out of cardboard. The height from the base to the tip of the smaller rocket is 20 cm. The height from the base to the tip of the larger rocket is 120 cm. What percent of the area of the smaller rocket is the area of the larger rocket?**



Height of smaller rocket: 20 cm

Height of larger rocket: 120 cm

Scale factor:

$$\begin{aligned}\text{Quantity} &= \text{Percent} \times \text{Whole}\\ \text{Actual height of larger rocket} &= \text{Percent} \times \text{height of smaller rocket}\\ 120 &= \text{Percent} \times 20\\ 6 &= \text{Percent}\\ 600\%&\end{aligned}$$

Area of larger rocket:

$$\begin{aligned}&(\text{scale factor})^2(\text{area of smaller rocket})\\ &(6)^2(\text{area of smaller rocket})\\ &36A\\ &36 = 36 \times 100\% = 3600\%\end{aligned}$$

The area of the larger rocket is $3,600\%$ *the area of the smaller rocket.*

4. **In the photo frame depicted below, three 5 inch by 5 inch squares are cut out for photographs. If these cut-out regions make up $\frac{3}{16}$ of the area of the entire photo frame, what are the dimensions of the photo frame?**

Since the cut-out regions make up $\frac{3}{16}$ *of the entire photo frame, then each cut-out region makes up* $\frac{\frac{3}{16}}{3} = \frac{1}{16}$ *of the entire photo frame.*

The relationship between the area of the scale drawing is

$(\text{square factor})^2 \times \text{area of original drawing}.$

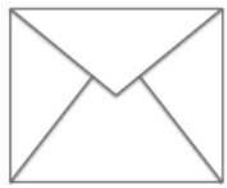
The area of each cut-out is $\frac{1}{16}$ *of the area of the original photo frame. Therefore, the square of the scale factor is* $\frac{1}{16}$*. Since* $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$*, the scale factor that relates the cut-out to the entire photo frame is* $\frac{1}{4}$*, or* 25%*.*

To find the dimensions of the square photo frame:

$$\begin{aligned}\text{Quantity} &= \text{Percent} \times \text{Whole}\\ \text{Small square side length} &= \text{Percent} \times \text{Photo frame side length}\\ 5 \text{ in.} &= 25\% \times \text{Photo frame side length}\\ 5 \text{ in.} &= \frac{1}{4} \times \text{Photo frame side length}\\ 4(5) \text{ in.} &= 4\left(\frac{1}{4}\right) \times \text{Photo frame side length}\\ 20 \text{ in.} &= \text{Photo frame side length}\end{aligned}$$

The dimensions of the square photo frame are 20 in. *by* 20 in.

5. Kelly was online shopping for envelopes for party invitations and saw these images on a website.


The website listed the dimensions of the small envelope as 6 in. by 8 in. and the medium envelope as 10 in. by $13\frac{1}{3}$ in.

a. Compare the dimensions of the small and medium envelopes. If the medium envelope is a scale drawing of the small envelope, what is the scale factor?

To find the scale factor,

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$\text{Medium height} = \text{Percent} \times \text{small height}$$
$$10 = \text{Percent} \times 6$$
$$\frac{10}{6} = \frac{5}{3} = \frac{5}{3} \times 100\% = 166\frac{2}{3}\%$$

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$\text{Medium width} = \text{Percent} \times \text{Small width}$$
$$13\frac{1}{3} = \text{Percent} \times 8$$
$$\frac{13\frac{1}{3}}{8} = \frac{5}{3} = \frac{5}{3} \times 100\% = 166\frac{2}{3}\%$$

b. If the large envelope was created based on the dimensions of the small envelope using a scale factor of 250%, find the dimensions of the large envelope.

Scale factor is 250%, so multiply each dimension of the small envelope by 2.50.

Large envelope dimensions are as follows:

$$6(2.5) \text{ in.} = 15 \text{ in.} \qquad 8(2.5) \text{ in.} = 20 \text{ in.}$$

c. If the medium envelope was created based on the dimensions of the large envelope, what scale factor was used to create the medium envelope?

Scale factor:

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$\text{Medium} = \text{Percent} \times \text{Large}$$
$$10 = \text{Percent} \times 15$$
$$\frac{10}{15} = \text{Percent}$$
$$\frac{2}{3} = \frac{2}{3} \times 100\% = 66\frac{2}{3}\%$$

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$\text{Medium} = \text{Percent} \times \text{Large}$$
$$13\frac{1}{3} = \text{Percent} \times 20$$
$$\frac{13\frac{1}{3}}{20} = \text{Percent}$$
$$\frac{2}{3} = \frac{2}{3} \times 100\% = 66\frac{2}{3}\%$$

d. What percent of the area of the larger envelope is the area of the medium envelope?

Scale factor of larger to medium: $66\frac{2}{3}\% = \frac{2}{3}$

Area: $\left(\frac{2}{3}\right)^2 = \frac{4}{9} = \frac{4}{9} \times 100\% = 44\frac{4}{9}\%$

The area of the medium envelope is $44\frac{4}{9}\%$ of the larger envelope.