

## Conditions for a Unique Triangle—Two Angles and a

### Given Side

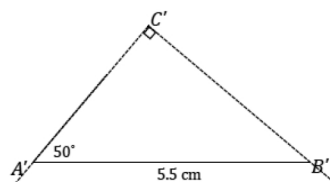
1. \_\_\_\_\_ has angles \_\_\_\_\_ and \_\_\_\_\_ and side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition. Under what condition is \_\_\_\_\_ drawn? Use your construction to explain why \_\_\_\_\_ is or is not identical to \_\_\_\_\_ .

2. \_\_\_\_\_ has angles \_\_\_\_\_ and \_\_\_\_\_ and side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition. Under what condition is \_\_\_\_\_ drawn? Use your construction to explain why \_\_\_\_\_ is or is not identical to \_\_\_\_\_ .

1. In triangle \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_. Side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition as \_\_\_\_\_. Leave all construction marks as evidence of your work, and label all side and angle measurements. What can be concluded about \_\_\_\_\_ and \_\_\_\_\_? Justify your response.
2. In triangle \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_. Side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition as \_\_\_\_\_. Leave all construction marks as evidence of your work, and label all side and angle measurements. What can be concluded about \_\_\_\_\_ and \_\_\_\_\_? Justify your response.
3. \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ are collinear, and \_\_\_\_\_. What can be concluded about \_\_\_\_\_ and \_\_\_\_\_? Justify your answer.
4. Draw \_\_\_\_\_ so that \_\_\_\_\_ has a measurement of \_\_\_\_\_, \_\_\_\_\_ has a measurement of \_\_\_\_\_, and \_\_\_\_\_ has a length of \_\_\_\_\_ cm. What are the lengths of the other sides?
5. Draw \_\_\_\_\_ so that \_\_\_\_\_ has a measurement of \_\_\_\_\_, \_\_\_\_\_ has a measurement of \_\_\_\_\_, and \_\_\_\_\_ has a length of \_\_\_\_\_ cm. What is the length of the longest side?

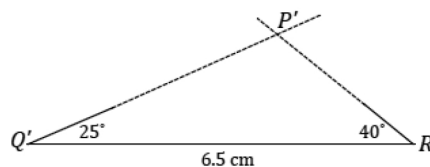
1. \_\_\_\_\_ has angles \_\_\_\_\_ and \_\_\_\_\_ and side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition. Under what condition is \_\_\_\_\_ drawn? Use your construction to explain why \_\_\_\_\_ is or is not identical to \_\_\_\_\_.

The condition on \_\_\_\_\_ is the two angles and the side opposite a given angle condition. Triangle \_\_\_\_\_ is identical to \_\_\_\_\_. After drawing the given side length, I used the protractor to draw the angle adjacent to \_\_\_\_\_. I drew the angle opposite the given side, \_\_\_\_\_, on a slip of paper and lined up one ray of the angle on patty paper with one ray of the angle adjacent to the given side. I moved the angle on patty paper along the coinciding rays until the free ray just met the endpoint of \_\_\_\_\_. There is no other way to draw this triangle; therefore, \_\_\_\_\_ must be identical to \_\_\_\_\_.

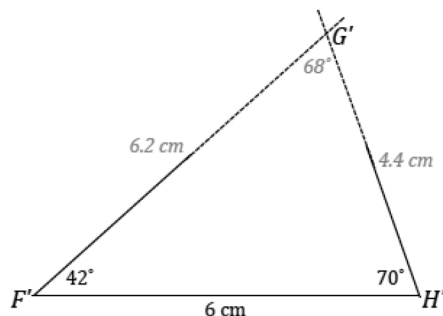


2. \_\_\_\_\_ has angles \_\_\_\_\_ and \_\_\_\_\_ and side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition. Under what condition is \_\_\_\_\_ drawn? Use your construction to explain why \_\_\_\_\_ is or is not identical to \_\_\_\_\_.

The condition on \_\_\_\_\_ is the two angles and included side condition. Triangle \_\_\_\_\_ is identical to \_\_\_\_\_. After drawing the given side length, I used the protractor to draw the angle adjacent to \_\_\_\_\_. After drawing the included side length, I used the protractor to draw the provided angle measurements at either endpoint of the included side \_\_\_\_\_. Since these two angle measurements are fixed, the two remaining side lengths will intersect in one location, which is the third vertex of the triangle, \_\_\_\_\_. There is no other way to draw this triangle; therefore, \_\_\_\_\_ must be identical to \_\_\_\_\_.



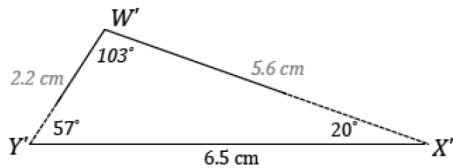
1. In triangle \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_. Side \_\_\_\_\_ cm. Draw triangle \_\_\_\_\_ under the same condition as \_\_\_\_\_. Leave all construction marks as evidence of your work, and label all side and angle measurements. What can be concluded about \_\_\_\_\_ and \_\_\_\_\_? Justify your response.



\_\_\_\_\_ and \_\_\_\_\_ are identical triangles by the two angles and included side condition. Since both triangles are drawn under the same condition, and the two angles and included side condition determines a unique triangle, both triangles determine the same unique triangle. Therefore, they are identical.

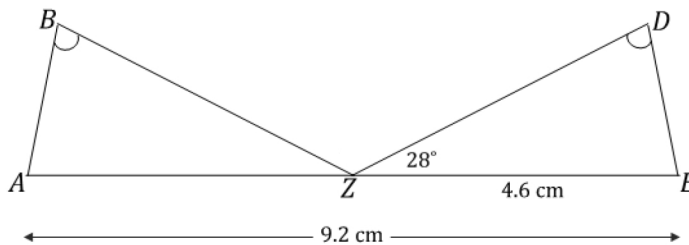
2. In triangle  $\triangle WY'X'$ ,  $\angle Y' = 57^\circ$ ,  $\angle X' = 20^\circ$ , and side  $WY' = 2.2$  cm. Draw triangle  $\triangle W'Y'X'$  under the same condition as  $\triangle WY'X'$ . Leave all construction marks as evidence of your work, and label all side and angle measurements.

What can be concluded about  $\triangle WY'X'$  and  $\triangle W'Y'X'$ ? Justify your response.



$\triangle WY'X'$  and  $\triangle W'Y'X'$  are identical triangles by the two angles and the side opposite a given angle condition. Since both triangles are drawn under the same condition, and the two angles and the side opposite a given angle condition determines a unique triangle, both triangles determine the same unique triangle. Therefore, they are identical.

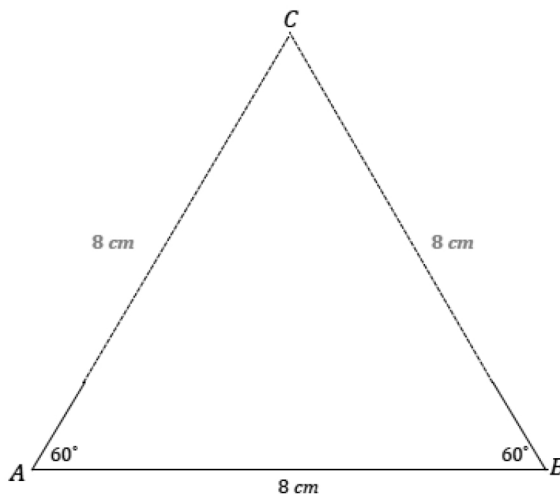
3.  $A$ ,  $Z$ , and  $E$  are collinear, and  $BZ \parallel DE$ . What can be concluded about  $\triangle ABZ$  and  $\triangle EDZ$ ? Justify your answer.



$\triangle ABZ$  and  $\triangle EDZ$  are identical by the two angles and the side opposite a given angle condition. Since segments  $AZ$  and  $ZE$  add, and  $AZ$  is  $4.6$  cm and  $ZE$  is  $4.6$  cm,  $AE$  must be  $9.2$  cm. Since angles on a line sum to  $180^\circ$ , and  $\angle BZE = 28^\circ$ , then  $\angle ABZ = \angle EDZ$ . From the diagram, we can see that  $\angle BAZ = \angle DEZ$ . The same measurements in both triangles satisfy the two angles and the side opposite a given angle condition, which means they both determine the same unique triangle; thus, they are identical.

4. Draw  $\triangle ABC$  so that  $\angle A = 60^\circ$ ,  $\angle B = 60^\circ$ , and  $AB = 8$  cm. What are the lengths of the other sides?

Both of the other side lengths are  $8$  cm.



5. Draw  $\triangle ABC$  so that  $\angle A$  has a measurement of  $30^\circ$ ,  $\angle B$  has a measurement of  $60^\circ$ , and  $AC$  has a length of  $8.66$  cm. What is the length of the longest side?

*The longest side has a length of  $10$  cm.*

