Unique Triangles—Two Sides and a Non-Included Angle

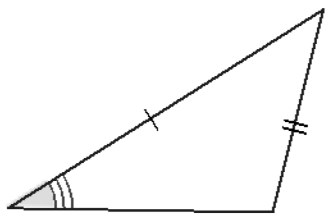
So far, we have learned about four conditions that determine unique triangles: three sides, two sides and an included angle, two angles and an included side, and two angles and the side opposite a given angle.

a. In this lesson, we studied the criterion two sides and a non-included angle. Which case of this criterion determines a unique triangle?

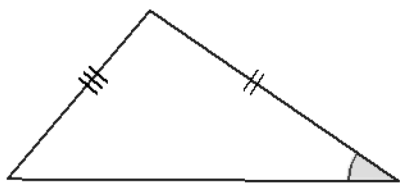
b. Provided has length cm, has length cm, and the measurement of is , draw , and describe why these conditions do not determine a unique triangle.

1. In each of the triangles below, two sides and a non-included acute angle are marked. Use a compass to draw a non-identical triangle that has the same measurements as the marked angle and marked sides (look at Exercise 1, part (e) of the Exploratory Challenge as a reference). Draw the new triangle on top of the old triangle. What is true about the marked angles in each triangle that results in two non-identical triangles under this condition?

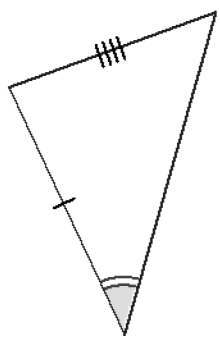
a.



b.

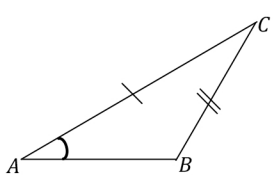


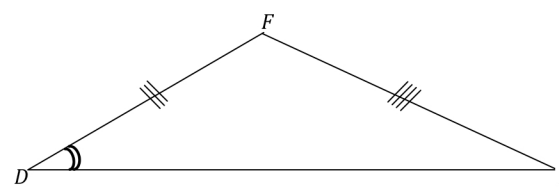
c.



2. Sometimes two sides and a non-included angle of a triangle determine a unique triangle, even if the angle is acute. In the following two triangles, copy the marked information (i.e., two sides and a non-included acute angle), and discover which determines a unique triangle. Measure and label the marked parts.

   In each triangle, how does the length of the marked side adjacent to the marked angle compare with the length of the side opposite the marked angle? Based on your drawings, specifically state when the two sides and acute non-included angle condition determines a unique triangle.





3. A sub-condition of the two sides and non-included angle is provided in each row of the following table. Decide whether the information determines a unique triangle. Answer with a "yes," "no," or "maybe" (for a case that may or may not determine a unique triangle).

	Condition	Determines a Unique Triangle?
1	Two sides and a non-included angle.	
2	Two sides and an acute, non-included angle.	
3	Two sides and a non-included angle.	
4	Two sides and a non-included angle, where the side adjacent to the angle is shorter than the side opposite the angle.	
5	Two sides and a non-included angle.	
6	Two sides and a non-included angle, where the side adjacent to the angle is longer than the side opposite the angle.	

4. Choose one condition from the chart in Problem 3 that does not determine a unique triangle, and explain why.

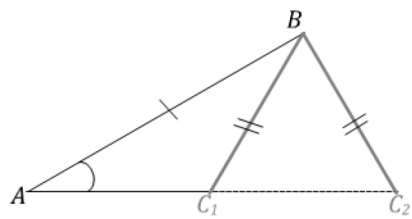
5. Choose one condition from the chart in Problem 3 that does determine a unique triangle, and explain why.

So far, we have learned about four conditions that determine unique triangles: three sides, two sides and an included angle, two angles and an included side, and two angles and the side opposite a given angle.

a. **In this lesson, we studied the criterion two sides and a non-included angle. Which case of this criterion determines a unique triangle?**

For the criterion two sides and a non-included angle, the case where the non-included angle is or greater determines a unique triangle.

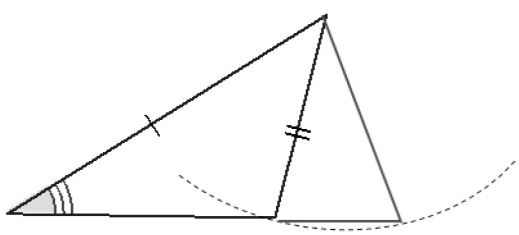
b. **Provided has length cm, has length cm, and the measurement of is , draw , and describe why these conditions do not determine a unique triangle.**



The non-included angle is an acute angle, and two different triangles can be determined in this case since can be in two different positions, forming a triangle with two different lengths of .

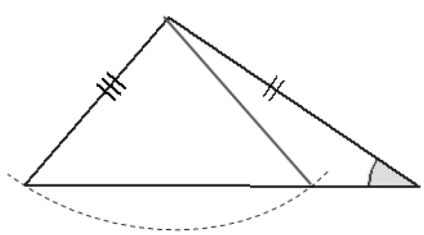
1. **In each of the triangles below, two sides and a non-included acute angle are marked. Use a compass to draw a non-identical triangle that has the same measurements as the marked angle and marked sides (look at Exercise 1, part (e) of the Exploratory Challenge as a reference). Draw the new triangle on top of the old triangle. What is true about the marked angles in each triangle that results in two non-identical triangles under this condition?**

a.



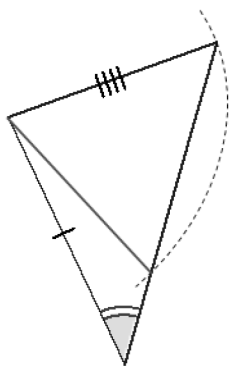
The non-included angle is acute.

b.



The non-included angle is acute.

c.



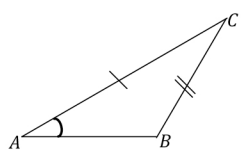
The non-included angle is acute.

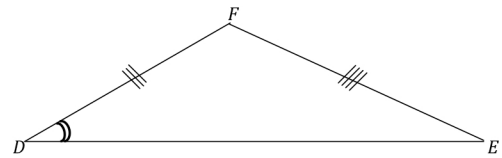
2. **Sometimes two sides and a non-included angle of a triangle determine a unique triangle, even if the angle is acute. In the following two triangles, copy the marked information (i.e., two sides and a non-included acute angle), and discover which determines a unique triangle. Measure and label the marked parts.**

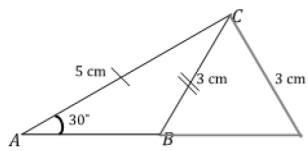
   In each triangle, how does the length of the marked side adjacent to the marked angle compare with the length of the side opposite the marked angle? Based on your drawings, specifically state when the two sides and acute non-included angle condition determines a unique triangle.

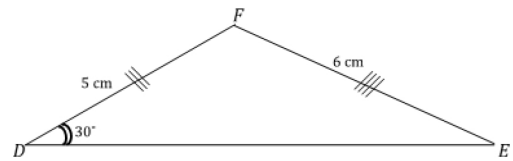
   While redrawing , students will see that a unique triangle is not determined, but in redrawing , a unique triangle is determined. In , the length of the side opposite the angle is shorter than the side adjacent to the angle. However, in , the side opposite the angle is longer than the side adjacent to the angle.

   The two sides and acute non-included angle condition determines a unique triangle if the side opposite the angle is longer than the side adjacent to the angle.









3. **A sub-condition of the two sides and non-included angle is provided in each row of the following table. Decide whether the information determines a unique triangle. Answer with a "yes," "no," or "maybe" (for a case that may or may not determine a unique triangle).**

	Condition	Determines a Unique Triangle?
1	Two sides and a non-included angle.	*yes*
2	Two sides and an acute, non-included angle.	*maybe*
3	Two sides and a non-included angle.	*yes*
4	Two sides and a non-included angle, where the side adjacent to the angle is shorter than the side opposite the angle.	*yes*
5	Two sides and a non-included angle.	*maybe*
6	Two sides and a non-included angle, where the side adjacent to the angle is longer than the side opposite the angle.	*no*

4. **Choose one condition from the chart in Problem 3 that does not determine a unique triangle, and explain why.**

   Possible response: Condition 6 does not determine a unique triangle because the condition of two sides and an acute non-included angle determines two possible triangles.

5. **Choose one condition from the chart in Problem 3 that does determine a unique triangle, and explain why.**

   Possible response: Condition 1 determines a unique triangle because the condition of two sides and a non-included angle with a measurement of or more determines a unique triangle.