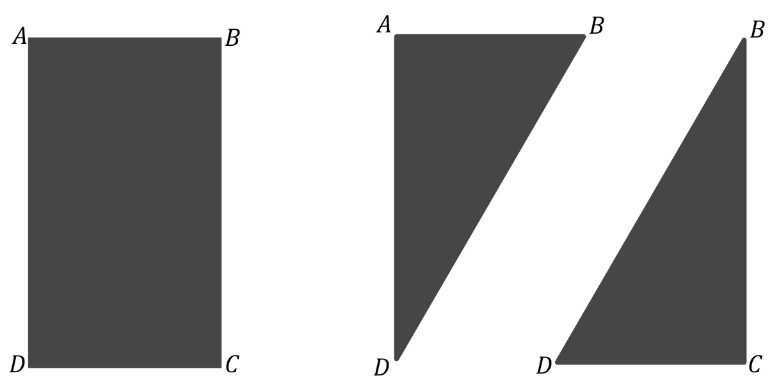
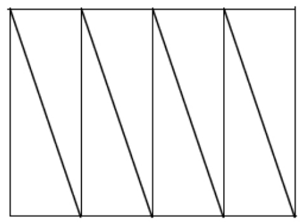
Using Unique Triangles to Solve Real-World and Mathematical Problems

Alice is cutting wrapping paper to size to fit a package. How should she cut the rectangular paper into two triangles to ensure that each piece of wrapping paper is the same? Use your knowledge of conditions that determine unique triangles to justify that the resulting pieces from the cut are the same.

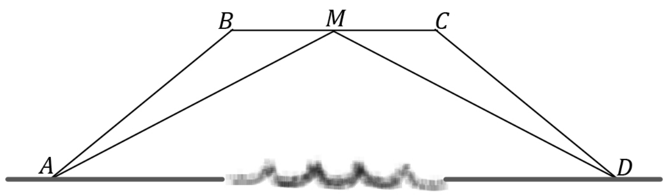


1. Jack is asked to cut a cake into equal pieces. He first cuts it into equal fourths in the shape of rectangles, and then he cuts each rectangle along a diagonal.

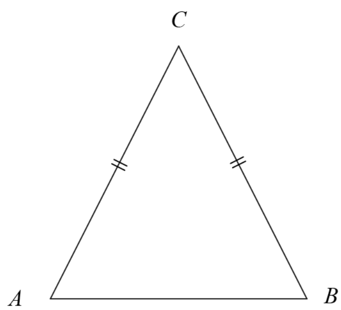
   Did he cut the cake into equal pieces? Explain.



2. The bridge below, which crosses a river, is built out of two triangular supports. The point lies on segment . The beams represented by and are equal in length, and the beams represented by and are equal in length. If the supports were constructed so that and are equal in measurement, is point the midpoint of ? Explain.

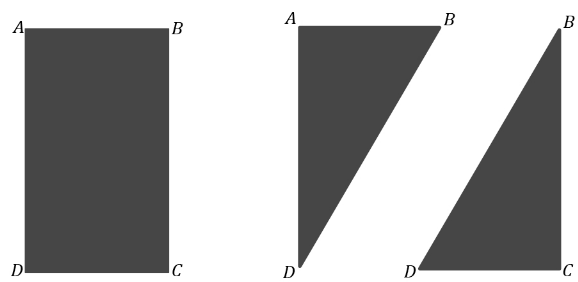


3. In , . Bill says that triangle correspondence matches three equal sides and shows that . Is Bill correct?



4. In the previous problem, Jill says that triangle correspondence matches two equal sides and the included angle. This also shows that . Is Jill correct?

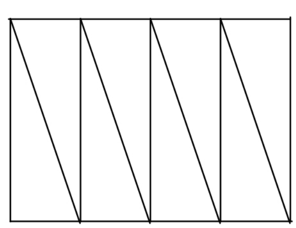
Alice is cutting wrapping paper to size to fit a package. How should she cut the rectangular paper into two triangles to ensure that each piece of wrapping paper is the same? Use your knowledge of conditions that determine unique triangles to prove that the resulting pieces from the cut are the same.

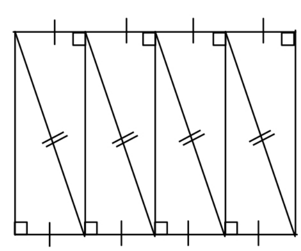


Alice should cut along the diagonal of rectangle . Since is a rectangle, the opposite sides will be equal in length, or, and . A rectangle also has four right angles, which means a cut along the diagonal will result in each triangle with one angle. The correspondence matches two equal pairs of sides and an equal, included pair of angles; the triangles are identical by the two sides and included angle condition.

1. **Jack is asked to cut a cake into equal pieces. He first cuts it into equal fourths in the shape of rectangles, and then he cuts each rectangle along a diagonal.**

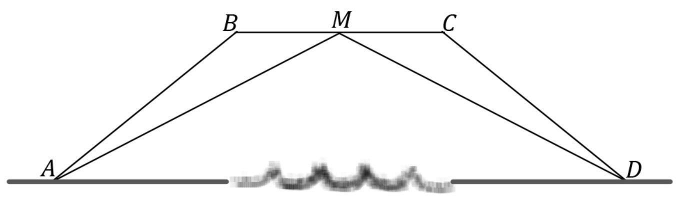
   Did he cut the cake into equal pieces? Explain.





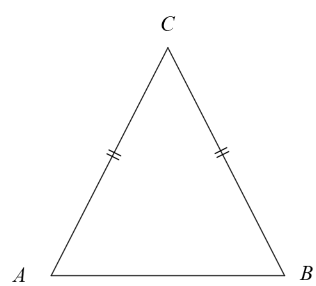
Yes, Jack cut the cake into equal pieces. Since the first series of cuts divided the cake into equal fourths in the shape of rectangles, we know that the opposite sides of the rectangles are equal in length; that means all triangles have two sides that are equal in length to each other. Each of the triangular pieces also has one right angle because we know that rectangles have four right angles. Therefore, there is a correspondence between all triangles that matches two pairs of equal sides and an equal, non-included angle, determining identical pieces of cake.

2. **The bridge below, which crosses a river, is built out of two triangular supports. The point lies on segment . The beams represented by and are equal in length, and the beams represented by and are equal in length. If the supports were constructed so that and are equal in measurement, is point the midpoint of ? Explain.**



Yes, is the midpoint of . The triangles are identical by the two sides and included angle condition. The correspondence matches two pairs of equal sides and one pair of included, equal angles. Since the triangles are identical, we can use the correspondence to conclude that , which makes the midpoint, by definition.

3. **In , . Bill says that triangle correspondence matches three equal sides and shows that . Is Bill correct?**



We are told that . The correspondence matches three equal sides: , , and ; this means is identical to by the three sides condition. From the correspondence, we can conclude that ; therefore, Bill is correct.

4. **In the previous problem, Jill says that triangle correspondence matches two equal sides and the included angle. This also shows that . Is Jill correct?**

We are told that . The correspondence matches two equal sides and the included angle: , , and ; this means is identical to by the two sides and included angle condition. From the correspondence, we can conclude that ; therefore, Jill is correct.