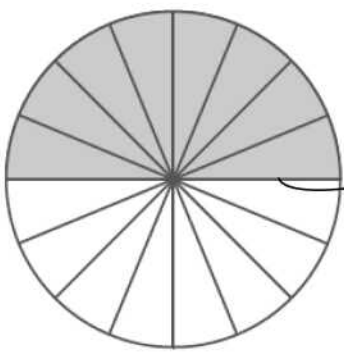
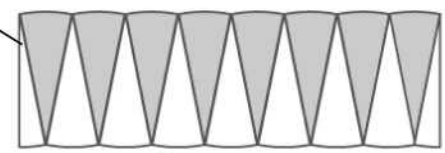
Name _____ Date_____

The Area of a Circle

Complete each statement using the words or algebraic expressions listed in the word bank below.





1. The length of the _____of the rectangular region approximates the length of the _____ of the circle.

2. The _____ of the rectangle approximates the length as one-half of the circumference of the circle.

3. The circumference of the circle is _____.

4. The _____ of the _____ is $2r$.

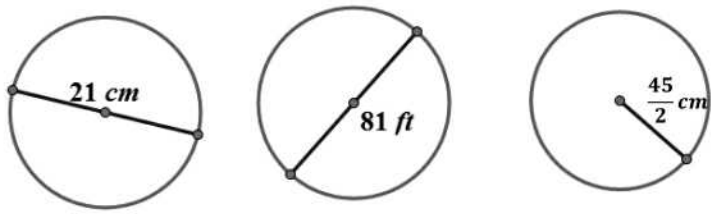
5. The ratio of the circumference to the diameter is _____.

6. Area (circle) = Area of (_____) = $\frac{1}{2} \cdot \text{circumference} \cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r =$ _____.

Word bank

Radius	**Height**	**Base**	$2\pi r$	
Diameter	**Circle**	**Rectangle**	πr^2	π

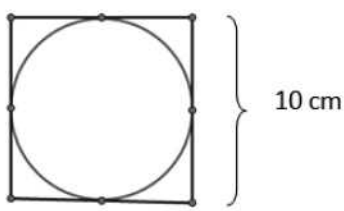
1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



2. A circle has a diameter of 20 inches.
   a. Find the exact area and find an approximate area using $\pi \approx 3.14$.
   b. What is the circumference of the circle using $\pi \approx 3.14$?

3. A circle has a diameter of 11 inches.
   a. Find the exact area and an approximate area using $\pi \approx 3.14$.
   b. What is the circumference of the circle using $\pi \approx 3.14$?

4. Using the figure below, find the area of the circle.



5. A path bounds a circular lawn at a park. If the inner edge of the path is 132 ft. around, approximate the amount of area of the lawn inside the circular path. Use $\pi \approx \frac{22}{7}$.

6. The area of a circle is 36π cm^2. Find its circumference.

7. Find the ratio of the area of two circles with radii 3 cm and 4 cm.

8. If one circle has a diameter of 10 cm and a second circle has a diameter of 20 cm, what is the ratio of the area of the larger circle to the area of the smaller circle?

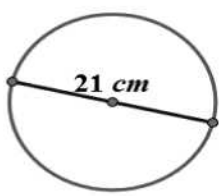
9. Describe a rectangle whose perimeter is 132 ft. and whose area is less than 1 ft^2. Is it possible to find a circle whose circumference is 132 ft. and whose area is less than 1 ft^2? If not, provide an example or write a sentence explaining why no such circle exists.

10. If the diameter of a circle is double the diameter of a second circle, what is the ratio of area of the first circle to the area of the second?

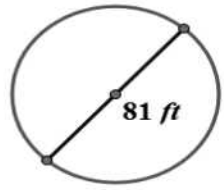
Complete each statement using the words or algebraic expressions listed in the word bank below.

1. The length of the *height* of the rectangular region approximates the length of the *radius* of the circle.
2. The *base* of the rectangle approximates the length as *one-half of the circumference* of the circle.
3. The *circumference* of the circle is $2\pi r$.
4. The *diameter* of the circle is $2r$.
5. The ratio of the circumference to the diameter is π.
6. Area (circle) = Area of (*rectangle*) $= \frac{1}{2} \cdot \text{circumference} \cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r = \pi r^2$.

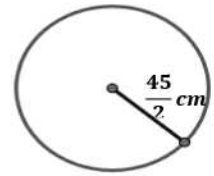
1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



346.5 cm^2



$5,155.1 \text{ ft}^2$



$1,591.1 \text{ cm}^2$

2. A circle has a diameter of 20 inches.
   a. Find the exact area and find an approximate area using $\pi \approx 3.14$.

   If the diameter is 20 in., *then the radius is* 10 in. *If* $A = \pi r^2$, *then* $A = \pi \cdot (10 \text{ in.})^2$ *or* $100\pi \text{ in}^2$.
   $A \approx (100 \cdot 3.14) \text{ in}^2 \approx 314 \text{ in}^2$.

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 20 in., *then the circumference is* $C = \pi d$ *or* $C \approx 3.14 \cdot 20 \text{ in.} \approx 62.8 \text{ in.}$

3. A circle has a diameter of 11 inches.
   a. Find the exact area and an approximate area using $\pi \approx 3.14$.

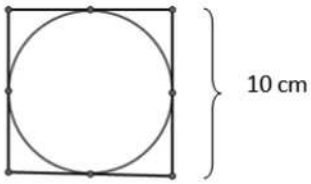
   If the diameter is 11 in., *then the radius is* $\frac{11}{2}$ *in. If* $A = \pi r^2$, *then* $A = \pi \cdot \left(\frac{11}{2} \text{ in.}\right)^2$ *or* $\frac{121}{4}\pi \text{ in}^2$.

   $$A \approx \left(\frac{121}{4} \cdot 3.14\right) \text{in}^2 \approx 94.985 \text{ in}^2$$

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 11 *inches, then the circumference is* $C = \pi d$ *or* $C \approx 3.14 \cdot 11 \text{ in.} \approx 34.54 \text{ in.}$

4. **Using the figure below, find the area of the circle.**



In this circle, the diameter is the same as the length of the side of the square. The diameter is 10 cm*; so, the radius is* 5 cm. $A = \pi r^2$, *so* $A = \pi(5 \text{ cm})^2 = 25\pi \text{ cm}^2$.

5. **A path bounds a circular lawn at a park. If the inner edge of the path is** 132 ft. **around, approximate the amount of area of the lawn inside the circular path. Use** $\pi \approx \frac{22}{7}$.

The length of the path is the same as the circumference. Find the radius from the circumference; then, find the area.

$$\begin{aligned} C &= 2\pi r \\ 132 \text{ ft.} &\approx 2 \cdot \frac{22}{7} \cdot r \\ 132 \text{ ft.} &\approx \frac{44}{7} r \\ \frac{7}{44} \cdot 132 \text{ ft.} &\approx \frac{7}{44} \cdot \frac{44}{7} r \\ 21 \text{ ft.} &\approx r \\ A &\approx \frac{22}{7} \cdot (21 \text{ ft.})^2 \\ A &\approx 1386 \text{ ft}^2 \end{aligned}$$

6. **The area of a circle is** $36\pi \text{ cm}^2$. **Find its circumference.**

Find the radius from the area of the circle; then, use it to find the circumference.

$$\begin{aligned} A &= \pi r^2 \\ 36\pi \text{ cm}^2 &= \pi r^2 \\ \frac{1}{\pi} \cdot 36\pi \text{ cm}^2 &= \frac{1}{\pi} \cdot \pi r^2 \\ 36 \text{ cm}^2 &= r^2 \\ 6 \text{ cm} &= r \\ C &= 2\pi r \\ C &= 2\pi \cdot 6 \text{ cm} \\ C &= 12\pi \text{ cm} \end{aligned}$$

7. **Find the ratio of the area of two circles with radii** 3 cm **and** 4 cm.

The area of the circle with radius 3 cm *is* $9\pi \text{ cm}^2$. *The area of the circle with the radius* 4 cm *is* $16\pi \text{ cm}^2$. *The ratio of the area of the two circles is* $\frac{9\pi \text{ cm}^2}{16\pi \text{ cm}^2}$ *or* $\frac{9}{16}$.

8. **If one circle has a diameter of** 10 cm **and a second circle has a diameter of** 20 cm, **what is the ratio of the area of the larger circle to the area of the smaller circle?**

The area of the circle with the diameter of 10 cm *will have a radius of* 5 cm. *The area of the circle with the diameter of* 10 cm *is* $\pi \cdot (5 \text{ cm})^2$ *or* $25\pi \text{ cm}^2$. *The area of the circle with the diameter of* 20 cm *will have a radius of* 10 cm. *The area of the circle with the diameter of* 20 cm *is* $\pi \cdot (10 \text{ cm})^2$ *or* $100\pi \text{ cm}^2$. *The ratio of the diameters is* 20 *to* 10 *or* $2:1$, *while the ratio of the areas is* $100\pi \text{ cm}^2$ *to* $25\pi \text{ cm}^2$ *or* $4:1$.

9. Describe a rectangle whose perimeter is 132 ft. and whose area is less than 1 ft^2. Is it possible to find a circle whose circumference is 132 ft. and whose area is less than 1 ft^2? If not, provide an example or write a sentence explaining why no such circle exists.

A rectangle that has a perimeter of 132 ft. can have a length of 65.995 ft. and a width of 0.005 ft. The area of such a rectangle is 0.329975 ft^2, which is less than 1 ft^2. No, because a circle that has a circumference of 132 ft. will have a radius of approximately 21 ft.

$A = \pi r^2 = \pi(21)^2 = 1387.96 \neq 1$

10. If the diameter of a circle is double the diameter of a second circle, what is the ratio of the area of the first circle to the area of the second?

If I choose a diameter of 24 cm for the first circle, then the diameter of the second circle is 12 cm. The first circle has a radius of 12 cm and an area of 144π cm^2. The second circle has a radius of 6 cm and an area of 36π cm^2. The ratio of the area of the first circle to the second is 144π cm^2 to 36π cm^2, which is a 4 to 1 ratio. The ratio of the diameters is 2, while the ratio of the areas is the square of 2, or 4.