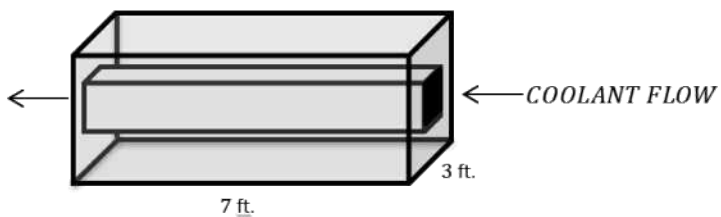
Name _____ Date _____

Volume and Surface Area

Lawrence is designing a cooling tank that is a square prism. A pipe in the shape of a smaller 2 ft. × 2 ft. square prism passes through the center of the tank as shown in the diagram, through which a coolant will flow.



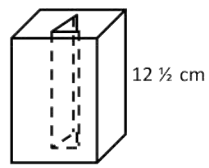
a. What is the volume of the tank including the cooling pipe?

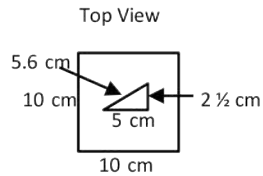
b. What is the volume of coolant that fits inside the cooling pipe?

c. What is the volume of the shell (the tank not including the cooling pipe)?

d. Find the surface area of the cooling pipe.

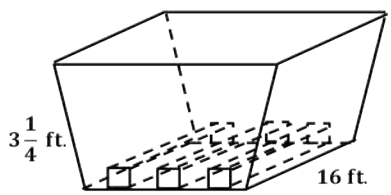
1. A child's toy is constructed by cutting a right triangular prism out of a right rectangular prism.

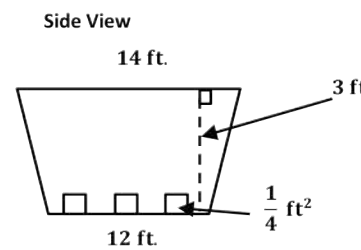




   a. Calculate the volume of the rectangular prism.
   b. Calculate the volume of the triangular prism.
   c. Calculate the volume of the material remaining in the rectangular prism.
   d. What is the largest number of triangular prisms that can be cut from the rectangular prism?
   e. What is the surface area of the triangular prism (assume there is no top or bottom)?

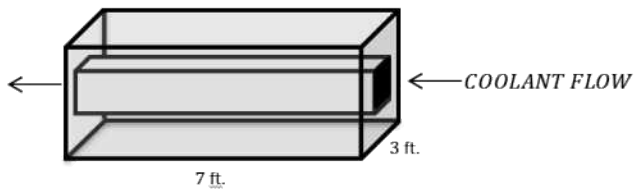
2. A landscape designer is constructing a flower bed in the shape of a right trapezoidal prism. He needs to run three identical square prisms through the bed for drainage.





   a. What is the volume of the bed without the drainage pipes?
   b. What is the total volume of the three drainage pipes?
   c. What is the volume of soil that can fit in the bed once the pipes are in place, assuming the amount of soil is filled to $\frac{3}{4}$ of the height of the planter?
   d. What is the height of the soil? If necessary, round to the nearest tenth.
   e. If the bed is made of 8 ft. $\times$ 4 ft. pieces of plywood, how many pieces of plywood will the landscape designer need to construct the bed without the drainage pipes?
   f. If the plywood needed to construct the bed costs $\$35$ per 8 ft. $\times$ 4 ft. piece, the drainage pipes cost $\$125$ each, and the soil costs $\$1.25$/cubic foot, how much does it cost to construct and fill the bed?

Lawrence is designing a cooling tank that is a square prism. A pipe in the shape of a smaller 2 ft. $\times$ 2 ft. square prism passes through the center of the tank as shown in the diagram, through which a coolant will flow.



Scaffolding:

If students have mastered this concept easily, assign only parts (c) and (d).

a. What is the volume of the tank including the cooling pipe?

$7 \text{ ft.} \times 3 \text{ ft.} \times 3 \text{ ft.} = 63 \text{ ft}^3$

b. What is the volume of coolant that fits inside the cooling pipe?

$2 \text{ ft.} \times 2 \text{ ft.} \times 7 \text{ ft.} = 28 \text{ ft}^3$

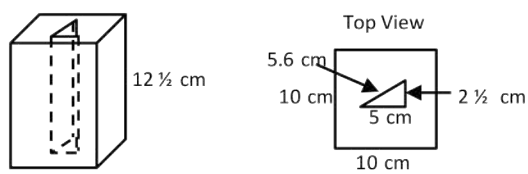
c. What is the volume of the shell (the tank not including the cooling pipe)?

$63 \text{ ft}^3 - 28 \text{ ft}^3 = 35 \text{ ft}^3$

d. Find the surface area of the cooling pipe.

$2 \text{ ft.} \times 7 \text{ ft.} \times 4 = 56 \text{ ft}^2$

1. A child's toy is constructed by cutting a right triangular prism out of a right rectangular prism.



a. Calculate the volume of the rectangular prism.

$10 \text{ cm} \times 10 \text{ cm} \times 12\frac{1}{2} \text{ cm} = 1250 \text{ cm}^3$

b. Calculate the volume of the triangular prism.

$\frac{1}{2}\left(5 \text{ cm} \times 2\frac{1}{2} \text{ cm}\right) \times 12\frac{1}{2} \text{ cm} = 78\frac{1}{8} \text{ cm}^3$

c. Calculate the volume of the material remaining in the rectangular prism.

$1250 \text{ cm}^3 - 78\frac{1}{8} \text{ cm}^3 = 1171\frac{7}{8} \text{ cm}^3$

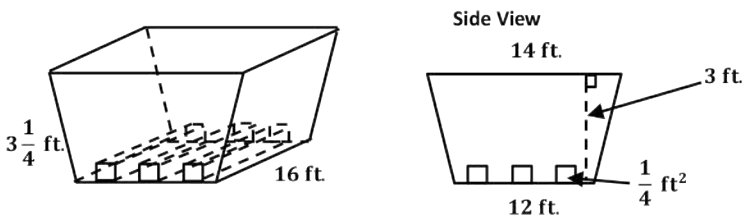
d. What is the largest number of triangular prisms that can be cut from the rectangular prism?

$$\frac{1250\ \text{cm}^3}{78\frac{1}{8}\ \text{cm}^3} = 16$$

e. What is the surface area of the triangular prism (assume there is no top or bottom)?

$$5.6\ \text{cm} \times 12\frac{1}{2}\ \text{cm} + 2\frac{1}{2}\ \text{cm} \times 12\frac{1}{2}\ \text{cm} + 5\ \text{cm} \times 12\ \frac{1}{2}\ \text{cm} = 163\frac{3}{4}\ \text{cm}^2$$

2. A landscape designer is constructing a flower bed in the shape of a right trapezoidal prism. He needs to run three identical square prisms through the bed for drainage.



a. What is the volume of the bed without the drainage pipes?

$$\frac{1}{2}(14\ \text{ft.} + 12\ \text{ft.}) \times 3\ \text{ft.} \times 16\ \text{ft.} = 624\ \text{ft}^3$$

b. What is the total volume of the three drainage pipes?

$$3\left(\frac{1}{4}\ \text{ft}^2 \times 16\ \text{ft.}\right) = 12\ \ \text{ft}^3$$

c. What is the volume of soil that can fit in the bed once the pipes are in place, assuming the amount of soil is filled to $\frac{3}{4}$ of the height of the planter?

$$\frac{3}{4}(624\ \text{ft}^3) - 12\ \text{ft}^3 = 456\ \text{ft}^3 \text{ or } \left[\frac{1}{2}(14\ \text{ft.} + 12\ \text{ft.}) \times \frac{3}{4}(3\ \text{ft.}) \times 16\ \text{ft.}\right] - 12\ \text{ft}^3 = 456\ \text{ft}^3$$

d. What is the height of the soil? If necessary, round to the nearest tenth.

$$\frac{456\ \text{ft}^3}{\frac{1}{2}(14\ \text{ft.} + 12\ \text{ft.}) \times 16\ \text{ft.}} \approx 2.2\ \text{ft. or } \frac{3}{4}(3\ \text{ft.}) = 2.25\ \text{ft.}$$

e. If the bed is made of $8\ \text{ft.} \times 4\ \text{ft.}$ pieces of plywood, how many pieces of plywood will the landscape designer need to construct the bed without the drainage pipes?

$$2\left(3\frac{1}{4}\ \text{ft.} \times 16\ \text{ft.}\right) + 12\ \text{ft.} \times 16\ \text{ft.} + 2\left(\frac{1}{2}(12\ \text{ft.} + 14\ \text{ft.}) \times 3\ \text{ft.}\right) = 374\ \text{ft}^2$$

$$374\ \text{ft}^2 \div \frac{(8\ \text{ft.} \times 4\ \text{ft.})}{\text{piece of plywood}} = 11.7, \text{ or } 12 \text{ pieces of plywood}$$

f. If the plywood needed to construct the bed costs $\$35$ per $8\ \text{ft.} \times 4\ \text{ft.}$ piece, the drainage pipes cost $\$125$ each, and the soil costs $\$1.25$/cubic foot, how much does it cost to construct and fill the bed?

$$\frac{\$35}{\text{piece of plywood}}(12 \text{ pieces of plywood}) + \frac{\$125}{\text{pipe}}(3 \text{ pipes}) + \frac{\$1.25}{\text{ft}^3 \text{ soil}}(456\ \text{ft}^3 \text{ soil}) = \$1,365.00$$