Name ______________________________________________ Date__________________

# Converting Rational Numbers to Decimals Using Long Division

1. What is the decimal value of $\frac{4}{11}$?

2. How do you know that $\frac{4}{11}$ is a repeating decimal?

3. What causes a repeating decimal in the long division algorithm?

1. Convert each rational number into its decimal form.

$\frac{1}{9} =$ ________________

$\frac{1}{6} =$ ________________

$\frac{2}{9} =$ ________________

$\frac{1}{3} =$ ________________ $\frac{2}{6} =$ ________________ $\frac{3}{9} =$ ________________

$\frac{4}{9} =$ ________________

$\frac{3}{6} =$ ________________

$\frac{5}{9} =$ ________________

$\frac{2}{3} =$ ________________ $\frac{4}{6} =$ ________________ $\frac{6}{9} =$ ________________

$\frac{7}{9} =$ ________________

$\frac{5}{6} =$ ________________

$\frac{8}{9} =$ ________________

One of these decimal representations is not like the others. Why?

**Enrichment:**

2. Chandler tells Aubrey that the decimal value of $-\frac{1}{17}$ is not a repeating decimal. Should Aubrey believe him? Explain.

3. Complete the quotients below without using a calculator and answer the questions that follow.

   a. Convert each rational number in the table to its decimal equivalent.

| | | | | |
|---|---|---|---|---|
| $\frac{1}{11} =$ | $\frac{2}{11} =$ | $\frac{3}{11} =$ | $\frac{4}{11} =$ | $\frac{5}{11} =$ |
| $\frac{6}{11} =$ | $\frac{7}{11} =$ | $\frac{8}{11} =$ | $\frac{9}{11} =$ | $\frac{10}{11} =$ |

   Do you see a pattern? Explain.

   b. Convert each rational number in the table to its decimal equivalent.

| | | | | |
|---|---|---|---|---|
| $\frac{0}{99} =$ | $\frac{10}{99} =$ | $\frac{20}{99} =$ | $\frac{30}{99} =$ | $\frac{45}{99} =$ |
| $\frac{58}{99} =$ | $\frac{62}{99} =$ | $\frac{77}{99} =$ | $\frac{81}{99} =$ | $\frac{98}{99} =$ |

   Do you see a pattern? Explain.

   c. Can you find other rational numbers that follow similar patterns?

1. What is the decimal value of $\frac{4}{11}$?

$\frac{4}{11} = 0.\overline{36}$

2. How do you know that $\frac{4}{11}$ is a repeating decimal?

*The prime factor in the denominator is $11$. Fractions that correspond with terminating decimals have only factors $2$ and $5$ in the denominator in simplest form.*

3. What causes a repeating decimal in the long division algorithm?

*When a remainder repeats, the division algorithm takes on a cyclic pattern causing a repeating decimal.*

1. Convert each rational number into its decimal form.

| | | |
|---|---|---|
| | | $\frac{1}{9} = 0.\overline{1}$ |
| | $\frac{1}{6} = 0.1\overline{6}$ | |
| | | $\frac{2}{9} = 0.\overline{2}$ |
| $\frac{1}{3} = 0.\overline{3}$ | $\frac{2}{6} = 0.\overline{3}$ | $\frac{3}{9} = 0.\overline{3}$ |
| | | $\frac{4}{9} = 0.\overline{4}$ |
| | $\frac{3}{6} = 0.5$ | |
| | | $\frac{5}{9} = 0.\overline{5}$ |
| $\frac{2}{3} = 0.\overline{6}$ | $\frac{4}{6} = 0.\overline{6}$ | $\frac{6}{9} = 0.\overline{6}$ |
| | | $\frac{7}{9} = 0.\overline{7}$ |
| | $\frac{5}{6} = 0.8\overline{3}$ | |
| | | $\frac{8}{9} = 0.\overline{8}$ |

One of these decimal representations is not like the others. Why?

*$\frac{3}{6}$ in its simplest form is $\frac{1}{2}$ (the common factor of $3$ divides out, leaving a denominator of $2$, which in decimal form will terminate.*

**Enrichment:**

**2.** **Chandler tells Aubrey that the decimal value of $-\frac{1}{17}$ is not a repeating decimal. Should Aubrey believe him? Explain.**

*No, Aubrey should not believe Chandler. The divisor $17$ is a prime number containing no factors of $2$ or $5$, and therefore, cannot be written as a terminating decimal. By long division, $-\frac{1}{17} = -0.\overline{0588235294117647}$; The decimal appears as though it is not going to take on a repeating pattern because all $16$ possible non-zero remainders appear before the remainder repeats. The seventeenth step produces a repeat remainder causing a cyclical decimal pattern.*

**3.** **Complete the quotients below without using a calculator and answer the questions that follow.**

**a.** **Convert each rational number in the table to its decimal equivalent.**

| | | | | |
|---|---|---|---|---|
| $\frac{1}{11} = 0.\overline{09}$ | $\frac{2}{11} = 0.\overline{18}$ | $\frac{3}{11} = 0.\overline{27}$ | $\frac{4}{11} = 0.\overline{36}$ | $\frac{5}{11} = 0.\overline{45}$ |
| $\frac{6}{11} = 0.\overline{54}$ | $\frac{7}{11} = 0.\overline{63}$ | $\frac{8}{11} = 0.\overline{72}$ | $\frac{9}{11} = 0.\overline{81}$ | $\frac{10}{11} = 0.\overline{90}$ |

**Do you see a pattern? Explain.**

*The two digits that repeat in each case have a sum of nine. As the numerator increases by one, the first of the two digits increases by one as the second of the digits decreases by one.*

**b.** **Convert each rational number in the table to its decimal equivalent.**

| | | | | |
|---|---|---|---|---|
| $\frac{0}{99} = 0$ | $\frac{10}{99} = 0.\overline{10}$ | $\frac{20}{99} = 0.\overline{20}$ | $\frac{30}{99} = 0.\overline{30}$ | $\frac{45}{99} = 0.\overline{45}$ |
| $\frac{58}{99} = 0.\overline{58}$ | $\frac{62}{99} = 0.\overline{62}$ | $\frac{77}{99} = 0.\overline{7}$ | $\frac{81}{99} = 0.\overline{81}$ | $\frac{98}{99} = 0.\overline{98}$ |

**Do you see a pattern? Explain.**

*The $2$-digit numerator in each fraction is the repeating pattern in the decimal form.*

**c.** **Can you find other rational numbers that follow similar patterns?**

*Answers will vary.*