Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

In a laboratory experiment, two mice will be placed in a simple maze with one decision point where a mouse can turn either left (L) or right (R). When the first mouse arrives at the decision point, the direction it chooses is recorded. Then, the process is repeated for the second mouse.

1. Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse. Determine all four possible decision outcomes for the two mice.

2. If the probability of turning left is 0.5, and the probability of turning right is 0.5 for each mouse, what is the probability that only one of the two mice will turn left?

3. If the researchers add food in the simple maze such that the probability of each mouse turning left is now 0.7, what is the probability that only one of the two mice will turn left?

1. Imagine that a family of three (Alice, Bill, and Chester) plays bingo at home every night. Each night, the chance that any one of the three players will win is $\frac{1}{3}$.

   a. Using A for Alice wins, B for Bill wins, and C for Chester wins, develop a tree diagram that shows the nine possible outcomes for two consecutive nights of play.

   b. Is the probability that "Bill wins both nights" the same as the probability that "Alice wins the first night and Chester wins the second night"? Explain.

2. According to the Washington, D.C. Lottery's website for its Cherry Blossom Doubler instant scratch game, the chance of winning a prize on a given ticket is about 17%. Imagine that a person stops at a convenience store on the way home from work every Monday and Tuesday to buy a Scratcher ticket to play the game.

   (Source: http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx accessed May 27, 2013).

   a. Develop a tree diagram showing the four possible outcomes of playing over these two days. Call stage 1 "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.

   b. What is the chance that the player will not win on Monday but will win on Tuesday?

   c. What is the chance that the player will win at least once during the two-day period?

Image of Tetrahedral Die

Source: http://commons.wikimedia.org/wiki/File:4-sided_dice_250.jpg

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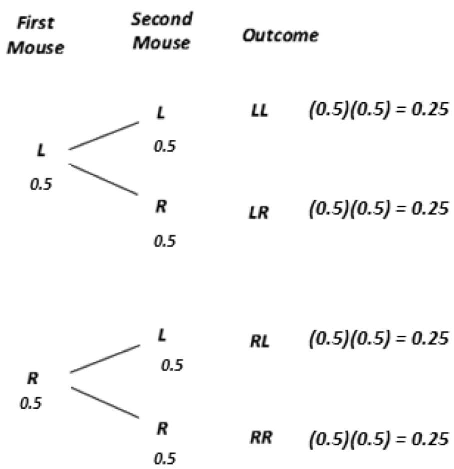
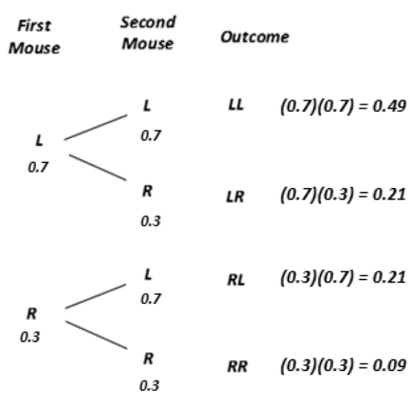
1. **Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse. Determine all four possible decision outcomes for the two mice.**



2. **If the probability of turning left is 0.5, and the probability of turning right is 0.5 for each mouse, what is the probability that only one of the two mice will turn left?**

   There are two outcomes that have exactly one mouse turning left: LR and RL. Each has a probability of 0.25, so the probability of only one of the two mice turning left is $0.25 + 0.25 = 0.5$.

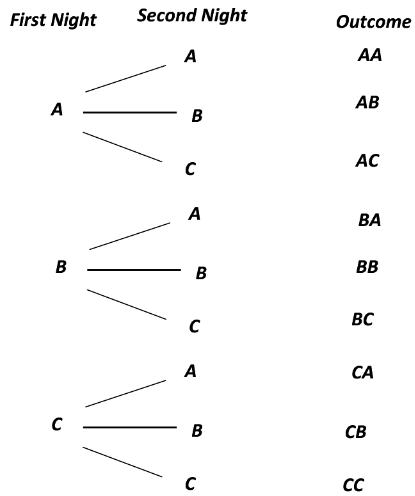
3. **If the researchers add food in the simple maze such that the probability of each mouse turning left is now 0.7, what is the probability that only one of the two mice will turn left?**



   As in Question 2, there are two outcomes that have exactly one mouse turning left: LR and RL. However, with the adjustment made by the researcher, each of these outcomes now has a probability of 0.21. So now, the probability of only one of the two mice turning left is $0.21 + 0.21 = 0.42$.

1. Imagine that a family of three (Alice, Bill, and Chester) plays bingo at home every night. Each night, the chance that any one of the three players will win is $\frac{1}{3}$.

   a. Using A for Alice wins, B for Bill wins, and C for Chester wins, develop a tree diagram that shows the nine possible outcomes for two consecutive nights of play.



   b. Is the probability that "Bill wins both nights" the same as the probability that "Alice wins the first night and Chester wins the second night"? Explain.

   Yes. The probability of Bill winning both nights is $\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{9}$, *which is the same as the probability of Alice winning the first night and Chester winning the second night (*$\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{9}$*).*

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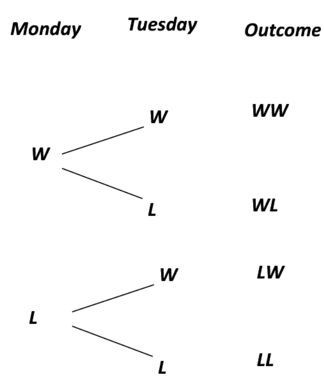
   a. Develop a tree diagram showing the four possible outcomes of playing over these two days. Call stage 1 "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.



b. **What is the chance that the player will not win on Monday but will win on Tuesday?**

LW outcome: $(\mathbf{0.83})(\mathbf{0.17}) = \mathbf{0.1411}$

c. **What is the chance that the player will win at least once during the two-day period?**

"Winning at least once" would include all outcomes except LL (which has a $\mathbf{0.6889}$ probability). The probabilities of these outcomes would sum to $\mathbf{0.3111}$.

Image of Tetrahedral Die

Source: http://commons.wikimedia.org/wiki/File:4-sided_dice_250.jpg

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