## **Calculating Probabilities of Compound Events**

In a laboratory experiment, three mice will be placed in a simple maze that has just one decision point where a mouse can turn either left (L) or right (R). When the first mouse arrives at the decision point, the direction he chooses is recorded. The same is done for the second and the third mice.

1. Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse, and so on. Determine all eight possible outcomes of the decisions for the three mice.

| 2. | Use the tree diagram from Question 1 to help answer the following question. If, for each mouse, the probability of turning left is 0.5 and the probability of turning right is 0.5, what is the probability that only one of the three mice will turn left?                           |
|----|---|
|    |   |
|    |   |
| 3. | If the researchers conducting the experiment add food in the simple maze such that the probability of each mouse turning left is now $0.7$ , what is the probability that only one of the three mice will turn left? To answer the question, use the tree diagram from Question $1$ . |
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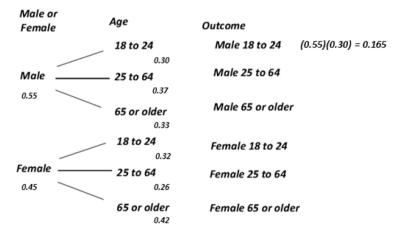
1. According to the Washington, D.C. Lottery's website for its Cherry Blossom Double" instant scratch game, the chance of winning a prize on a given ticket is about 17%. Imagine that a person stops at a convenience store on the way home from work every Monday, Tuesday, and Wednesday to buy a Scratcher ticket and plays the game.

(Source: <a href="http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx">http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx</a> accessed May 27, 2013)

- a. Develop a tree diagram showing the eight possible outcomes of playing over these three days. Call stage one "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.
- b. What is the probability that the player will not win on Monday but will win on Tuesday and Wednesday?
- c. What is the probability that the player will win at least once during the 3-day period?
- 2. A survey company is interested in conducting a statewide poll prior to an upcoming election. They are only interested in talking to registered voters.

Imagine that 55% of the registered voters in the state are male and 45% are female. Also, consider that the distribution of ages may be different for each group. In this state, 30% of male registered voters are age 18-24, 37% are age 25-64, and 33% are 65 or older. 32% of female registered voters are age 18-24, 26% are age 25-64, and 42% are 65 or older.

The following tree diagram describes the distribution of registered voters. The probability of selecting a male registered voter age 18-24 is 0.165.



- a. What is the chance that the polling company will select a registered female voter age 65 or older?
- b. What is the chance that the polling company will select any registered voter age 18–24?

In a laboratory experiment, three mice will be placed in a simple maze that has just one decision point where a mouse can turn either left (L) or right (R). When the first mouse arrives at the decision point, the direction he chooses is recorded. The same is done for the second and the third mice.

Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse, and so on. Determine all eight possible outcomes of the decisions for the three mice.

| First<br>Mouse | Second<br>Mouse | Third<br>Mouse   | Outcome |                         |
|----------------|-----------------|------------------|---------|-------------------------|
|                | L               | L<br>0.5         | Ш       | (0.5)(0.5)(0.5) = 0.125 |
| L              | 0.5             | R 0.5            | LLR     | (0.5)(0.5)(0.5) = 0.125 |
| 0.5            | R 0.5           | `                | LRL     | (0.5)(0.5)(0.5) = 0.125 |
|                |                 | R 0.5            | LRR     | (0.5)(0.5)(0.5) = 0.125 |
|                | , 1 3           | L<br>            | RLL     | (0.5)(0.5)(0.5) = 0.125 |
| R              | 0.5             | R <sub>0.5</sub> | RLR     | (0.5)(0.5)(0.5) = 0.125 |
| 0.5            | R <             | L<br>0.5         | RRL     | (0.5)(0.5)(0.5) = 0.125 |
|                | 0.5             | <b>R</b><br>0.5  | RRR     | (0.5)(0.5)(0.5) = 0.125 |

If the probability of turning left is 0.5 and the probability of turning right is 0.5 for each mouse, what is the probability that only one of the three mice will turn left?

There are three outcomes that have exactly one mouse turning left: LRR, RLR, and RRL. Each has a probability of 0.125, so the probability of having only one of the three mice turn left is 0.375.

3. If the researchers conducting the experiment add food in the simple maze such that the probability of each mouse turning left is now 0.7, what is the probability that only one of the three mice will turn left? To answer the question, use the tree diagram from Question 1.

As in Question 2, there are three outcomes that have exactly one mouse turning left: LRR, RLR, and RRL. However, with the adjustment made by the researcher, each of the three outcomes now has a probability of 0.063. So now, the probability of having only one of the three mice turn left is the sum of three equally likely outcomes of 0.063, or 0.063(3) = 0.189. The tree provides a way to organize the outcomes and the probabilities.

| First<br>Mouse | Second<br>Mouse | Third<br>Mouse   | Outcome                 |                         |
|----------------|-----------------|------------------|-------------------------|-------------------------|
|                | , L :           | L<br>            | Ш                       | (0.7)(0.7)(0.7) = 0.343 |
| L              | 0.7             | R <sub>0.3</sub> | LLR                     | (0.7)(0.7)(0.3) = 0.147 |
| 0.7            | R 0.7 0.7 R 0.3 | LRL              | (0.7)(0.3)(0.7) = 0.147 |                         |
|                |                 | R 0.3            | LRR                     | (0.7)(0.3)(0.3) = 0.063 |
|                | , , ;           | L<br>0.7         | RLL                     | (0.3)(0.7)(0.7) = 0.147 |
| R              | 0.7             | R 0.3            | RLR                     | (0.3)(0.7)(0.3) = 0.063 |
| 0.3            | R =             | L<br>0.7         | RRL                     | (0.3)(0.3)(0.7) = 0.063 |
|                |                 | R<br>0.3         | RRR                     | (0.3)(0.3)(0.3) = 0.027 |

According to the Washington, D.C. Lottery's website for its Cherry Blossom Doubler instant scratch game, the chance
of winning a prize on a given ticket is about 17%. Imagine that a person stops at a convenience store on the way
home from work every Monday, Tuesday, and Wednesday to buy a Scratcher ticket and plays the game.

(Source: http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx accessed May 27, 2013)

a. Develop a tree diagram showing the eight possible outcomes of playing over these three days. Call stage one "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.

| Monday | Tuesday | Wednesday | Outcome |
|--------|---------|-----------|---------|
|        |         | w         | www     |
| w      | _w <    | L         | WWL     |
|        | \       | w         | WLW     |
|        |         | L         | WLL     |
|        | —       | W         | LWW     |
|        | _ w _   | L         | LWL     |
|        |         | w         | LLW     |
|        |         | L         | LLL     |

b. What is the probability that the player will not win on Monday but will win on Tuesday and Wednesday?

LWW outcome: 0.83(0.17)(0.17) = 0.024

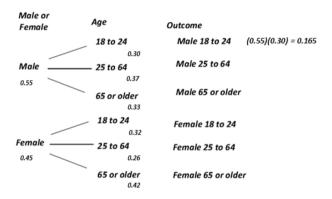
c. What is the probability that the player will win at least once during the 3-day period?

"Winning at least once" would include all outcomes except LLL (which has a 0.5718 probability). The probabilities of these outcomes would sum to about 0.4282. This is also equal to 1-0.5718.

A survey company is interested in conducting a statewide poll prior to an upcoming election. They are only interested in talking to registered voters.

Imagine that 55% of the registered voters in the state are male and 45% are female. Also, consider that the distribution of ages may be different for each group. In this state, 30% of male registered voters are age 18–24, 37% are age 25–64, and 33% are 65 or older. 32% of female registered voters are age 18–24, 26% are age 25–64, and 42% are 65 or older.

The following tree diagram describes the distribution of registered voters. The probability of selecting a male registered voter age 18-24 is 0.165.



a. What is the chance that the polling company will select a registered female voter age 65 or older?

Female 65 or older: (0.45)(0.42) = 0.189.

b. What is the chance that the polling company will select any registered voter age 18–24?

The probability of selecting any registered voter age 18–24 would be the sum of the probability of selecting a male registered voter age 18–24 and the probability of selecting a female registered voter age 18–24. Those values are (0.55)(0.30) = 0.165 and (0.45)(0.32) = 0.144, respectively.

So, the sum is 0.165 + 0.144 = 0.309.