Conducting a Simulation to Estimate the Probability of an Event

1. Nathan is your school's star soccer player. When he takes a shot on goal he typically scores half of the time. Suppose that he takes six shots in a game. To estimate the probability of the number of goals Nathan makes, use simulation with a number cube. One roll of a number cube represents one shot.
   a. Specify what outcome of a number cube you want to represent a goal scored by Nathan in one shot.

   b. For this problem, what represents a trial of taking six shots?

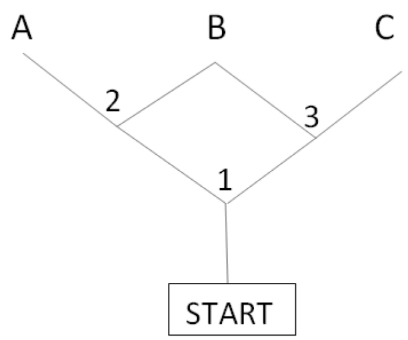
   c. Perform and list the results of ten trials of this simulation.

   d. Identify the number of goals Nathan made in each of the ten trials you did in part (c).

   e. Based on your ten trials, what is your estimate of the probability that Nathan scores three goals if he takes six shots in a game?

2. Suppose that Pat scores 40% of the shots he takes in a soccer game. If he takes six shots in a game, what would one simulated trial look like using a number cube in your simulation?

1. A mouse is placed at the start of the maze shown below. If it reaches station B, it is given a reward. At each point where the mouse has to decide which direction to go, assume that it is equally likely to go in either direction. At each decision point 1, 2, 3, it must decide whether to go left (L) or right (R). It cannot go backwards.



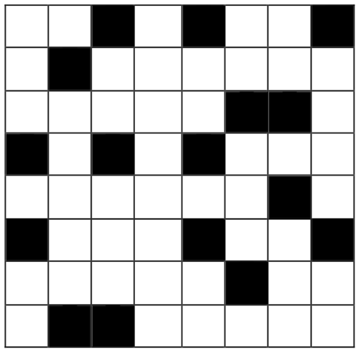
a. Create a theoretical model of probabilities for the mouse to arrive at terminal points A, B, and C.
   i. List the possible paths of a sample space for the paths the mouse can take. For example, if the mouse goes left at decision point 1, and then right at decision point 2, then the path would be denoted LR.
   ii. Are the paths in your sample space equally likely? Explain.
   iii. What are the theoretical probabilities that a mouse reaches terminal points A, B, and C? Explain.

b. Based on the following set of simulated paths, estimate the probabilities that the mouse arrives at points A, B, and C.

RR	RR	RL	LL	LR	RL	LR	LL	LR	RR
LR	RL	LR	RR	RL	LR	RR	LL	RL	RL
LL	LR	LR	LL	RR	RR	RL	LL	RR	LR
RR	LR	RR	LR	LR	LL	LR	RL	RL	LL

c. How do the simulated probabilities in part (b) compare to the theoretical probabilities of part (a)?

2. Suppose that a dartboard is made up of the 8×8 grid of squares shown below. Also, suppose that when a dart is thrown, it is equally likely to land on any one of the 64 squares. A point is won if the dart lands on one of the 16 black squares. Zero points are earned if the dart lands in a white square.



a. For one throw of a dart, what is the probability of winning a point? Note that a point is won if the dart lands on a black square.

b. Lin wants to use a number cube to simulate the result of one dart. She suggests that 1 on the number cube could represent a win. Getting $2, 3$, or 4 could represent no point scored. She says that she would ignore getting a 5 or 6. Is Lin's suggestion for a simulation appropriate? Explain why you would use it, or if not, how you would change it.

c. Suppose a game consists of throwing a dart three times. A trial consists of three rolls of the number cube. Based on Lin's suggestion in part (b) and the following simulated rolls, estimate the probability of scoring two points in three darts.

324	332	411	322	124
224	221	241	111	223
321	332	112	433	412
443	322	424	412	433
144	322	421	414	111
242	244	222	331	224
113	223	333	414	212
431	233	314	212	241
421	222	222	112	113
212	413	341	442	324

d. The theoretical probability model for winning 0, 1, 2, and 3 points in three throws of the dart as described in this problem is

i. Winning 0 points has a probability of 0.42;

ii. Winning 1 point has a probability of 0.42;

iii. Winning 2 points has a probability of 0.14;

iv. Winning 3 points has a probability of 0.02.

Use the simulated rolls in part (c) to build a model of winning $0, 1, 2$, and 3 points, and compare it to the theoretical model.

1. **Nathan is your school's star soccer player. When he takes a shot on goal he typically scores half of the time. Suppose that he takes six shots in a game. To estimate the probability of the number of goals Nathan makes, use simulation with a number cube. One roll of a number cube represents one shot.**

   a. **Specify what outcome of a number cube you want to represent a goal scored by Nathan in one shot.**

   Answers will vary; students need to determine which three numbers on the number cube represent scoring a goal.

   b. **For this problem, what represents a trial of taking six shots?**

   Rolling the cube six times represents taking six shots on goal or 1 simulated trial.

   c. **Perform and list the results of ten trials of this simulation.**

   Answers will vary; students working in pairs works well for these problems. Performing only ten trials is a function of time. Ideally, many more trials should be done. If there is time, have your class pool their results.

   d. **Identify the number of goals Nathan made in each of the ten trials you did in part (c).**

   Answers will vary.

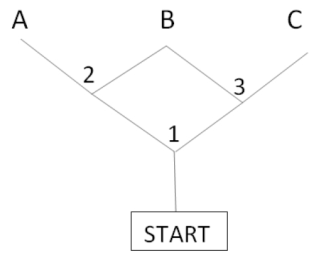
   e. **Based on your ten trials, what is your estimate of the probability that Nathan scores three goals if he takes six shots in a game?**

   Answers will vary; the probability of scoring per shot is $\frac{1}{2}$.

2. **Suppose that Pat scores 40% of the shots he takes in a soccer game. If he takes six shots in a game, what would one simulated trial look like using a number cube in your simulation?**

   Students need to realize that 40% is 2 out of 5. In order to use the number cube as the device, 1 and 2 could represent goals, while 3, 4, and 5 could represent missed shots, and 6 is ignored. Rolling the number cube six times creates 1 simulated trial.

1. **A mouse is placed at the start of the maze shown below. If it reaches station B, it is given a reward. At each point where the mouse has to decide which direction to go, assume that it is equally likely to go in either direction. At each decision point $1, 2, 3$, it must decide whether to go left (L) or right (R). It cannot go backwards.**



a. **Create a theoretical model of probabilities for the mouse to arrive at terminal points A, B, and C.**

i. **List the possible paths of a sample space for the paths the mouse can take. For example, if the mouse goes left at decision point 1, and then right at decision point 2, then the path would be denoted LR.**

The possible paths in the sample space are {LL, LR, RL, RR}.

ii. **Are the paths in your sample space equally likely? Explain.**

Each of these outcomes has an equal probability of $\frac{1}{4}$, since at each decision point there are only two possible choices, which are equally likely.

iii. **What are the theoretical probabilities that a mouse reaches terminal points A, B, and C? Explain.**

The probability of reaching terminal point A is $\frac{1}{4}$, since it is accomplished by path LL. Similarly, reaching terminal point C is $\frac{1}{4}$, since it is found by path RR. However, reaching terminal point B is $\frac{1}{2}$, since it is reached via LR or RL.

b. **Based on the following set of simulated paths, estimate the probabilities that the mouse arrives at points A, B, and C.**

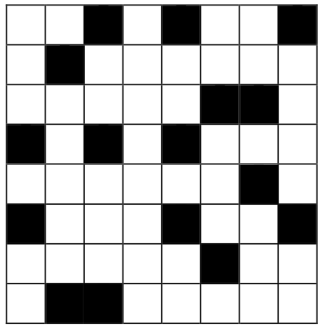
RR	**RR**	**RL**	**LL**	**LR**	**RL**	**LR**	**LL**	**LR**	**RR**
LR	**RL**	**LR**	**RR**	**RL**	**LR**	**RR**	**LL**	**RL**	**RL**
LL	**LR**	**LR**	**LL**	**RR**	**RR**	**RL**	**LL**	**RR**	**LR**
RR	**LR**	**RR**	**LR**	**LR**	**LL**	**LR**	**RL**	**RL**	**LL**

Students need to go through the list and count the number of paths that go to A, B, and C. They should find the estimated probabilities to be $\frac{8}{40} = 0.2$ for A, $\frac{22}{40} = 0.55$ for B, and $\frac{10}{40} = 0.25$ for C.

c. **How do the simulated probabilities in part (b) compare to the theoretical probabilities of part (a)?**

The probabilities are reasonably close for parts (a) and (b). Probabilities based on taking 400 trials should be closer than those based on 40, but the probabilities based on 40 are in the ballpark.

2. **Suppose that a dartboard is made up of the 8×8 grid of squares shown below. Also, suppose that when a dart is thrown, it is equally likely to land on any one of the 64 squares. A point is won if the dart lands on one of the 16 black squares. Zero points are earned if the dart lands in a white square.**



a. **For one throw of a dart, what is the probability of winning a point? Note that a point is won if the dart lands on a black square.**

The probability of winning a point is $\frac{16}{64} = 0.25$.

b. **Lin wants to use a number cube to simulate the result of one dart. She suggests that 1 on the number cube could represent a win. Getting 2, 3, or 4 could represent no point scored. She says that she would ignore getting a 5 or 6. Is Lin's suggestion for a simulation appropriate? Explain why you would use it, or if not, how you would change it.**

Lin correctly suggests that to simulate the result of one throw, a number cube could be used with the 1 representing a hit, 2, 3, 4 representing a missed throw, while ignoring 5 and 6. (As an aside, a tetrahedron could be used by using the side facing down as the result.)

c. **Suppose a game consists of throwing a dart three times. A trial consists of three rolls of the number cube. Based on Lin's suggestion in part (b) and the following simulated rolls, estimate the probability of scoring two points in three darts.**

324	332	411	322	124
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431	233	314	212	241
421	222	222	112	113
212	413	341	442	324

The probability of scoring two points in three darts is $\frac{5}{50} = 0.1$. *(Students need to count the number of trials that contain exactly two 1's.)*

d. The theoretical probability model for winning 0, 1, 2, and 3 points in three throws of the dart as described in this problem is

i. Winning 0 points has a probability of 0.42;

ii. Winning 1 point has a probability of 0.42;

iii. Winning 2 points has a probability of 0.14;

iv. Winning 3 points has a probability of 0.02.

Use the simulated rolls in part (c) to build a model of winning $0, 1, 2$, and 3 points, and compare it to the theoretical model.

To find the estimated probability of 0 points, count the number of trials that have no 1's in them ($\frac{23}{50} = 0.46$).

To find the estimated probability of 1 point, count the number of trials that have one 1 in them ($\frac{20}{50} = 0.4$).

From part (c), the estimated probability of 2 points is 0.1.

To find the estimated probability of 3 points, count the number of trials that have three 1's in them ($\frac{2}{50} = 0.04$).

The theoretical and simulated probabilities are reasonably close.

	0	1	2	3
Theoretical	0.42	0.42	0.14	0.02
Simulated	0.46	0.40	0.10	0.04