Name ____ Date_________________

Proofs of Laws of Exponents

1. Show directly that for any positive integer x, $x^{-5} \cdot x^{-7} = x^{-12}$.

2. Show directly that for any positive integer x, $x^{-2}\;{}^{-3} = x^{6}$.

1. You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? No friend received the photo twice. Express your answer in exponential notation.

# of New People to View Your Photo	*Total # of People to View Your Photo*

2. Show directly, without using (11), that $1.27^{-36\ \ 85} = 1.27^{-36\cdot 85}$.

3. Show directly that $\frac{2}{13}^{-127} \cdot \frac{2}{13}^{-56} = \frac{2}{13}^{-183}$.

4. Prove for any positive number x, $x^{-127} \cdot x^{-56} = x^{-183}$.

5. Prove for any positive number x, $x^{-m} \cdot x^{-n} = x^{-m-n}$ for positive integers m and n.

6. Which of the preceding four problems did you find easiest to do? Explain.

7. Use the properties of exponents to write an equivalent expression that is a product of distinct primes, each raised to an integer power.

$$\frac{10^5 \cdot 9^2}{6^4} =$$

1. **Show directly that for any positive integer x, $x^{-5} \cdot x^{-7} = x^{-12}$.**

$x^{-5} \cdot x^{-7} = \frac{1}{x^5} \cdot \frac{1}{x^7}$	*By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*
$= \frac{1}{x^5 \cdot x^7}$	*By the Product Formula for Complex Fractions*
$= \frac{1}{x^{5+7}}$	*By $x^m \cdot x^n = x^{m+n}$ for whole numbers m and n (6)*
$= \frac{1}{x^{12}}$	
$= x^{-12}$	*By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

2. **Show directly that for any positive integer x, $x^{-2 \quad -3} = x^6$.**

$x^{-2 \quad -3} = \frac{1}{x^{-2 \quad 3}}$	*By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*
$= \frac{1}{x^{- \ 2\cdot 3}}$	*By Case (ii) of (11)*
$= \frac{1}{x^{-6}}$	
$= x^6$	*By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

1. **You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? No friend received the photo twice. Express your answer in exponential notation.**

# of New People to View Your Photo	*Total # of People to View Your Photo*
7	7
5×7	$7 + 5 \times 7$
$5 \times 5 \times 7$	$7 + 5 \times 7 + 5^2 \times 7$
$5 \times 5 \times 5 \times 7$	$7 + 5 \times 7 + 5^2 \times 7 + 5^3 \times 7$

The total number of people who viewed the photo is $5^0 + 5^1 + 5^2 + 5^3 \quad \times 7$.

2. **Show directly, without using (11), that $1.27^{-36\ 85} = 1.27^{-36 \cdot 85}$.**

$$1.27^{-36\ 85} = \left(\frac{1}{1.27^{36}}\right)^{85}$$ *By definition.*

$$= \frac{1}{1.27^{36\ 85}}$$ *By $\left(\frac{1}{x}\right)^m = \frac{1}{x^m}$ for any whole number m (C)*

$$= \frac{1}{1.27^{36 \cdot 85}}$$ *By $(x^m)^n = x^{mn}$ for whole numbers m and n (7)*

$$= 1.27^{-36 \cdot 85}$$ *By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

3. **Show directly that $\left(\frac{2}{13}\right)^{-127} \cdot \left(\frac{2}{13}\right)^{-56} = \left(\frac{2}{13}\right)^{-183}$.**

$$\left(\frac{2}{13}\right)^{-127} \cdot \left(\frac{2}{13}\right)^{-56} = \frac{1}{\left(\frac{2}{13}\right)^{127}} \cdot \frac{1}{\left(\frac{2}{13}\right)^{56}}$$ *By definition*

$$= \frac{1}{\left(\frac{2}{13}\right)^{127} \cdot \left(\frac{2}{13}\right)^{56}}$$ *By the product formula for complex fractions*

$$= \frac{1}{\left(\frac{2}{13}\right)^{127+56}}$$ *By $x^m \cdot x^n = x^{m+n}$ for whole numbers m and n (6)*

$$= \frac{1}{\left(\frac{2}{13}\right)^{183}}$$

$$= \left(\frac{2}{13}\right)^{-183}$$ *By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

4. **Prove for any positive number x, $x^{-127} \cdot x^{-56} = x^{-183}$**

$$x^{-127} \cdot x^{-56} = \frac{1}{x^{127}} \cdot \frac{1}{x^{56}}$$ *By definition*

$$= \frac{1}{x^{127} \cdot x^{56}}$$ *By the product formula for complex fractions*

$$= \frac{1}{x^{127+56}}$$ *By $x^m \cdot x^n = x^{m+n}$ for whole numbers m and n (6)*

$$= \frac{1}{x^{183}}$$

$$= x^{-183}$$ *By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

5. **Prove for any positive number x, $x^{-m} \cdot x^{-n} = x^{-m-n}$ for positive integers m and n.**

$$x^{-m} \cdot x^{-n} = \frac{1}{x^m} \cdot \frac{1}{x^n}$$ *By definition*

$$= \frac{1}{x^m \cdot x^n}$$ *By the product formula for complex fractions*

$$= \frac{1}{x^{m+n}}$$ *By $x^m \cdot x^n = x^{m+n}$ for whole numbers m and n (6)*

$$= x^{-(m+n)}$$ *By $x^{-m} = \frac{1}{x^m}$ for any whole number m (B)*

$$= x^{-m-n}$$

6. **Which of the preceding four problems did you find easiest to do? Explain.**

Students will likely say that $x^{-m} \cdot x^{-n} = x^{-m-n}$ (Problem 5) was the easiest problem to do. It requires the least amount of writing because the symbols are easier to write than decimal or fraction numbers.

7. **Use the properties of exponents to write an equivalent expression that is a product of distinct primes, each raised to an integer power.**

$$\frac{\mathbf{10^5 \cdot 9^2}}{\mathbf{6^4}} = \frac{2 \cdot 5\ ^5 \cdot\ 3 \cdot 3\ ^2}{2 \cdot 3\ ^4} = \frac{2^5 \cdot 5^5 \cdot 3^2 \cdot 3^2}{2^4 \cdot 3^4} = 2^{5-4} \cdot 3^{4-4} \cdot 5^5 = 2^1 \cdot 3^0 \cdot 5^5 = 2^1 \cdot 1 \cdot 5^5 = 2 \cdot 5^5$$