Name ____ Date____________________

Multiplication of Numbers in Exponential Form

Simplify each of the following numerical expressions as much as possible:

1. Let a and b be positive integers. $23^a \times 23^b =$

2. $5^3 \times 25 =$

3. Let x and y be positive integers and $x > y$. $\frac{11^x}{11^y} =$

4. $\frac{2^{13}}{8} =$

1. A certain ball is dropped from a height of x feet. It always bounces up to $\frac{2}{3}x$ feet. Suppose the ball is dropped from 10 feet and is caught exactly when it touches the ground after the 30^{th} bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

Bounce	Computation of Distance Traveled in Previous Bounce	Total Distance Traveled (in feet)
1		
2		
3		
4		
30		
n		

2. If the same ball is dropped from 10 feet and is caught exactly at the highest point after the 25^{th} bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

3. Let a and b be numbers and $b \neq 0$, and let m and n be positive integers. Simplify each of the following expressions as much as possible:

$-19^{5} \cdot -19^{11} =$	$2.7^{5} \times 2.7^{3} =$
$\frac{7^{10}}{7^{3}} =$	$\frac{1}{5}^{2} \cdot \frac{1}{5}^{15} =$
$-\frac{9}{7}^{m} \cdot -\frac{9}{7}^{n} =$	$\frac{ab^{3}}{b^{2}} =$

4. Let the dimensions of a rectangle be $(4 \times 871209^{5} + 3 \times 49762105)$ ft. by $7 \times 871209^{3} - 49762105^{4}$ ft. Determine the area of the rectangle. No need to expand all the powers.

5. A rectangular area of land is being sold off in smaller pieces. The total area of the land is 2^{15} square miles. The pieces being sold are 8^{3} square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.

Note to Teacher: Accept both forms of the answer; in other words, the answer that shows the exponents as a sum or difference and the answer where the numbers were actually added or subtracted.

Simplify each of the following numerical expressions as much as possible:

1. **Let a and b be positive integers. $23^a \times 23^b =$**

   $23^a \times 23^b = 23^{a+b}$

2. **$5^3 \times 25 =$**

   $$5^3 \times 25 = 5^3 \times 5^2$$
   $$= 5^{3+2}$$
   $$= 5^5$$

3. **Let x and y be positive integers and $x > y$. $\frac{11^x}{11^y} =$**

   $\frac{11^x}{11^y} = 11^{x-y}$

4. **$\frac{2^{13}}{8} =$**

   $$\frac{2^{13}}{8} = \frac{2^{13}}{2^3}$$
   $$= 2^{13-3}$$
   $$= 2^{10}$$

To ensure success, students need to complete at least bounces 1–4 with support in class.

Students may benefit from a simple drawing of the scenario. It will help them see why the factor of 2 is necessary when calculating the distance traveled for each bounce. Make sure to leave the total distance traveled in the format shown so that students can see the pattern that is developing. Simplifying at any step will make it extremely difficult to write the general statement for n number of bounces.

1. A certain ball is dropped from a height of x feet. It always bounces up to $\frac{2}{3}x$ feet. Suppose the ball is dropped from 10 feet and is caught exactly when it touches the ground after the 30^{th} bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

Bounce	Computation of Distance Traveled in Previous Bounce	Total Distance Traveled (in feet)
1	$2 \ \frac{2}{3} \ 10$	$10 + 2 \ \frac{2}{3} \ 10$
2	$2 \ \frac{2}{3} \ \frac{2}{3} \ 10$ $= 2 \ \frac{2}{3}^{2} \ 10$	$10 + 2 \ \frac{2}{3} \ 10 + 2 \ \frac{2}{3}^{2} \ 10$
3	$2 \ \frac{2}{3} \ \frac{2}{3}^{2} \ 10$ $= 2 \ \frac{2}{3}^{3} \ 10$	$10 + 2 \ \frac{2}{3} \ 10 + 2 \ \frac{2}{3}^{2} \ 10 + 2 \ \frac{2}{3}^{3} \ 10$
4	$2 \ \frac{2}{3} \ \frac{2}{3}^{3} \ 10$ $= 2 \ \frac{2}{3}^{4} \ 10$	$10 + 2 \ \frac{2}{3} \ 10 + 2 \ \frac{2}{3}^{2} \ 10 + 2 \ \frac{2}{3}^{3} \ 10 + 2 \ \frac{2}{3}^{4} \ 10$
30	$2 \ \frac{2}{3}^{30} \ 10$	$10 + 2 \ \frac{2}{3} \ 10 + 2 \ \frac{2}{3}^{2} \ 10 + 2 \ \frac{2}{3}^{3} \ 10 + 2 \ \frac{2}{3}^{4} \ 10 + \cdots + 2 \ \frac{2}{3}^{30} \ 10$
n	$2 \ \frac{2}{3}^{n} \ 10$	$10 + 20 \ \frac{2}{3} \ 1 + \frac{2}{3} + \frac{2}{3}^{2} + \cdots + \frac{2}{3}^{n}$

2. If the same ball is dropped from 10 feet and is caught exactly at the highest point after the 25^{th} bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

Based on the last problem we know that each bounce causes the ball to travel $2 \ \frac{2}{3}^{n} \ 10$ feet. If the ball is caught at the highest point of the 25^{th} bounce, then the distance traveled on that last bounce is just $\frac{2}{3}^{25} \ 10$ because it does not make the return trip to the ground. Therefore, the total distance traveled by the ball in this situation is

$$10 + 2 \ \frac{2}{3} \ 10 + 2 \ \frac{2}{3}^{2} \ 10 + 2 \ \frac{2}{3}^{3} \ 10 + 2 \ \frac{2}{3}^{4} \ 10 + \cdots + 2 \ \frac{2}{3}^{23} \ 10 + 2 \ \frac{2}{3}^{24} \ 10$$

3. Let a and b be numbers and $b \neq 0$, and let m and n be positive integers. Simplify each of the following expressions as much as possible:

$(-19)^5 \cdot (-19)^{11} = (-19)^{5+11}$	$2.7^5 \times 2.7^3 = 2.7^{5+3}$
$\frac{7^{10}}{7^3} = 7^{10-3}$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} = \left(\frac{1}{5}\right)^{2+15}$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n = \left(-\frac{9}{7}\right)^{m+n}$	$\frac{ab^3}{b^2} = ab^{3-2}$

4. Let the dimensions of a rectangle be $(4 \times (871209)^5 + 3 \times 49762105)$ ft. by $(7 \times (871209)^3 - (49762105)^4)$ ft. Determine the area of the rectangle. No need to expand all the powers.

Area $= (4 \times (871209)^5 + 3 \times 49762105)(7 \times (871209)^3 - (49762105)^4)$

$= 28 \times (871209)^8 - 4 \times (871209)^5 (49762105)^4 + 21 \times (871209)^3 (49762105) - 3 \times (49762105)^5$ *sq. ft.*

5. A rectangular area of land is being sold off in smaller pieces. The total area of the land is 2^{15} square miles. The pieces being sold are 8^3 square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.

$8^3 = 2^9$

$\frac{2^{15}}{2^9} = 2^{15-9} = 2^6 = 64$

64 *pieces of land can be sold.*