Numbers in Exponential Form Raised to a Power

Write each answer as a base raised to a power or as the product of bases raised to powers that is equivalent to the given one.

1. $9^{3}\;{}^{6} =$

2. $113^{2} \times 37 \times 51^{4}\;{}^{3} =$

3. Let x, y, z be numbers. $x^{2}yz^{4}\;{}^{3} =$

4. Let x, y, z be numbers and let m, n, p, q be positive integers. $x^{m}y^{n}z^{p}\;{}^{q} =$

5. $\frac{4^{8}}{5^{8}} =$

1. Show (prove) in detail why $2 \cdot 3 \cdot 7\ ^{4} = 2^4 3^4 7^4$.

2. Show (prove) in detail why $xyz\ ^{4} = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $xyz\ ^{n} = x^n y^n z^n$ for any numbers x, y, and z and for any positive integer n.

Write each answer as a base raised to a power or as the product of bases raised to powers that is equivalent to the given one.

1. $(9^3)^6 =$

$(9^3)^6 = 9^{6\times 3} = 9^{18}$

2. $(113^2 \times 37 \times 51^4)^3 =$

$(113^2 \times 37 \times 51^4)^3 = ((113^2 \times 37) \times 51^4)^3$ *(associative law)*

$= (113^2 \times 37)^3 \times (51^4)^3$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= (113^2)^3 \times 37^3 \times (51^4)^3$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= 113^6 \times 37^3 \times 51^{12}$ *(because $(x^m)^n = x^{mn}$ for all numbers x)*

3. **Let x, y, z be numbers.** $(x^2yz^4)^3 =$

$(x^2yz^4)^3 = ((x^2 \times y) \times z^4)^3$ *(associative law)*

$= (x^2 \times y)^3 \times (z^4)^3$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= (x^2)^3 \times y^3 \times (z^4)^3$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= x^6 \times y^3 \times z^{12}$ *(because $(x^m)^n = x^{mn}$ for all numbers x)*

$= x^6y^3z^{12}$

4. **Let x, y, z be numbers and let m, n, p, q be positive integers.** $(x^my^nz^p)^q =$

$(x^my^nz^p)^q = ((x^m \times y^n) \times z^p)^q$ *(associative law)*

$= (x^m \times y^n)^q \times (z^p)^q$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= (x^m)^q \times (y^n)^q \times (z^p)^q$ *(because $(xy)^n = x^n y^n$ for all numbers x, y)*

$= x^{mp} \times y^{nq} \times z^{pq}$ *(because $(x^m)^n = x^{mn}$ for all numbers x)*

$= x^{mq}y^{nq}z^{pq}$

5. $\frac{4^8}{5^8} =$

$\frac{4^8}{5^8} = \left(\frac{4}{5}\right)^8$

1. **Show (prove) in detail why $2 \cdot 3 \cdot 7\ ^4 = 2^4 3^4 7^4$.**

$2 \cdot 3 \cdot 7\ ^4 =$	$2 \cdot 3 \cdot 7 \quad 2 \cdot 3 \cdot 7 \quad 2 \cdot 3 \cdot 7 \quad 2 \cdot 3 \cdot 7$	*By definition*
	$= \ 2 \cdot 2 \cdot 2 \cdot 2 \quad 3 \cdot 3 \cdot 3 \cdot 3 \quad 7 \cdot 7 \cdot 7 \cdot 7$	*By repeated use of the commutative and associative properties*
	$= 2^4 3^4 7^4$	*By definition*

2. **Show (prove) in detail why $xyz\ ^4 = x^4 y^4 z^4$ for any numbers x, y, z.**

The left side of the equation, $xyz\ ^4$*, means* $(xyz)\,(xyz)\,(xyz)\,(xyz)$*. Using the commutative and associative properties of multiplication, we can write* $(xyz)\,(xyz)\,(xyz)\,(xyz)$ *as* $(xxxx)(\,yyyy)\,(zzzz)$*, which in turn can be written as* $x^4 y^4 z^4$*, which is what the right side of the equation states.*

3. **Show (prove) in detail why $xyz\ ^n = x^n y^n z^n$ for any numbers x, y, and z and for any positive integer n.**

Beginning with the left side of the equation, $xyz\ ^n$ *means* $\underbrace{(xyz) \cdot (xyz) \cdots (xyz)}_{n\ times}$*. Using the commutative and associative properties of multiplication,* $\underbrace{(xyz) \cdot (xyz) \cdots (xyz)}_{n\ times}$ *can be rewritten as* $\underbrace{(x \cdot x \cdots x)}_{n\ times}\,\underbrace{(y \cdot y \cdots y)}_{n\ times}\,\underbrace{(z \cdot z \cdots z)}_{n\ times}$ *and finally,* $x^n y^n z^n$*, which is what the right side of the equation states. We can also prove this equality by a different method, as follows. Beginning with the right side,* $x^n y^n z^n$ *means* $\underbrace{(x \cdot x \cdots x)}_{n\ times}\,\underbrace{(y \cdot y \cdots y)}_{n\ times}\,\underbrace{(z \cdot z \cdots z)}_{n\ times}$*, which by the commutative property of multiplication can be rewritten as* $\underbrace{(xyz) \cdot (xyz) \cdots (xyz)}_{n\ times}$*. Using exponential notation,* $\underbrace{(xyz) \cdot (xyz) \cdots (xyz)}_{n\ times}$ *can be rewritten as* $xyz\ ^n$*, which is what the left side of the equation states.*