Name _____ Date _____

Solving Radical Equations

1. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$x^2 + 4x = 4(x + 16)$$

2. Find the positive value of x that makes the equation true, and then verify your solution is correct.

$$(4x)^3 = 1728$$

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1. $x^2(x+7) = \frac{1}{2}(14x^2+16)$

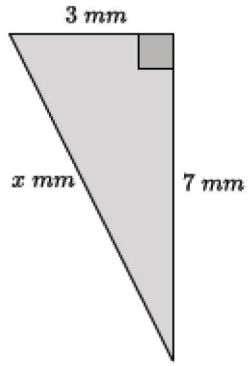
2. $x^3 = 1{,}331^{-1}$

3. $\frac{x^9}{x^7} - 49 = 0$. Determine the positive value of x that makes the equation true, and then explain how you solved the equation.

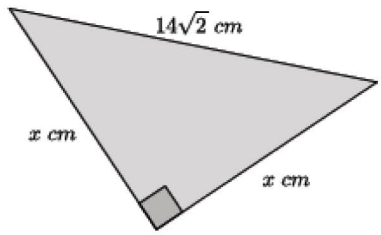
4. $(8x)^2 = 1$. Determine the positive value of $\boldsymbol{x}$ that makes the equation true.

5. $\left(9\sqrt{x}\right)^2 - 43x = 76$

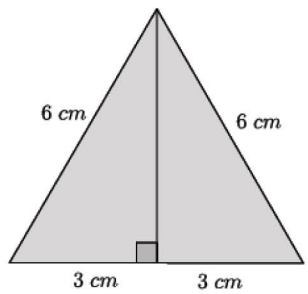
6. Determine the length of the hypotenuse of the right triangle below.



7. Determine the length of the legs in the right triangle below.



8. An equilateral triangle has side lengths of 6 cm. What is the height of the triangle? What is the area of the triangle?



9. Challenge: $\left(\frac{1}{2}x\right)^2 - 3x = 7x + 8 - 10x$. Find the positive value of $\boldsymbol{x}$ that makes the equation true.

10. Challenge: $11x + x(x - 4) = 7(x + 9)$. Find the positive value of $\boldsymbol{x}$ that makes the equation true.

1. **Find the positive value of x that makes the equation true, and then verify your solution is correct.**

$$x^2 + 4x = 4(x + 16)$$

$$\begin{aligned} x^2 + 4x &= 4(x + 16) \\ x^2 + 4x &= 4x + 64 \\ x^2 + 4x - 4x &= 4x - 4x + 64 \\ x^2 &= 64 \\ \sqrt{x^2} &= \sqrt{64} \\ x &= 8 \end{aligned}$$

Check:

$$\begin{aligned} 8^2 + 4(8) &= 4(8 + 16) \\ 64 + 32 &= 4(24) \\ 96 &= 96 \end{aligned}$$

2. **Find the positive value of x that makes the equation true, and then verify your solution is correct.**

$$(4x)^3 = 1728$$

$$\begin{aligned} (4x)^3 &= 1,728 \\ 64x^3 &= 1,728 \\ \frac{1}{64}(64x^3) &= (1,728)\frac{1}{64} \\ x^3 &= 27 \\ \sqrt[3]{x^3} &= \sqrt[3]{27} \\ x &= 3 \end{aligned}$$

Check:

$$\begin{aligned} (4 \times 3)^3 &= 1,728 \\ 12^3 &= 1,728 \\ 1,728 &= 1,728 \end{aligned}$$

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1. $x^2(x + 7) = \frac{1}{2}(14x^2 + 16)$

$$\begin{aligned} x^2(x + 7) &= \frac{1}{2}(14x^2 + 16) \\ x^3 + 7x^2 &= 7x^2 + 8 \\ x^3 + 7x^2 - 7x^2 &= 7x^2 - 7x^2 + 8 \\ x^3 &= 8 \\ \sqrt[3]{x^3} &= \sqrt[3]{8} \\ x &= 2 \end{aligned}$$

Check:

$$\begin{aligned} 2^2(2 + 7) &= \frac{1}{2}(14(2^2) + 16) \\ 4(9) &= \frac{1}{2}(56 + 16) \\ 36 &= \frac{1}{2}(72) \\ 36 &= 36 \end{aligned}$$

2. $x^3 = 1,331^{-1}$

$$x^3 = 1,331^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{1,331^{-1}}$$
$$x = \sqrt[3]{\frac{1}{1,331}}$$
$$x = \sqrt[3]{\frac{1}{11^3}}$$
$$x = \frac{1}{11}$$

Check:

$$\left(\frac{1}{11}\right)^3 = 1,331^{-1}$$
$$\frac{1}{11^3} = 1,331^{-1}$$
$$\frac{1}{1,331} = 1,331^{-1}$$
$$1,331^{-1} = 1,331^{-1}$$

3. $\frac{x^9}{x^7} - 49 = 0$. **Determine the positive value of x that makes the equation true, and then explain how you solved the equation.**

$$\frac{x^9}{x^7} - 49 = 0$$
$$x^2 - 49 = 0$$
$$x^2 - 49 + 49 = 0 + 49$$
$$x^2 = 49$$
$$\sqrt{x^2} = \sqrt{49}$$
$$x = 7$$

Check:

$$7^2 - 49 = 0$$
$$49 - 49 = 0$$
$$0 = 0$$

To solve the equation I first had to simplify the expression $\frac{x^9}{x^7}$ to x^2. Next, I used the properties of equality to transform the equation into $x^2 = 49$. Finally, I had to take the square root of both sides of the equation to solve for x.

4. $(8x)^2 = 1$. **Determine the positive value of x that makes the equation true.**

$$(8x)^2 = 1$$
$$64x^2 = 1$$
$$\sqrt{64x^2} = \sqrt{1}$$
$$8x = 1$$
$$\frac{8x}{8} = \frac{1}{8}$$
$$x = \frac{1}{8}$$

Check:

$$\left(8\left(\frac{1}{8}\right)\right)^2 = 1$$
$$1^2 = 1$$
$$1 = 1$$

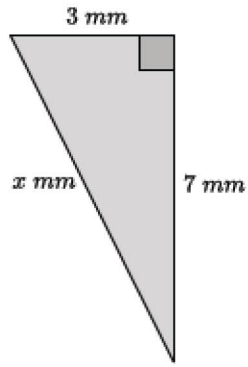
5. $\left(9\sqrt{x}\right)^2 - 43x = 76$.

$$\left(9\sqrt{x}\right)^2 - 43x = 76$$
$$9^2\left(\sqrt{x}\right)^2 - 43x = 76$$
$$81x - 43x = 76$$
$$38x = 76$$
$$\frac{38x}{38} = \frac{76}{38}$$
$$x = 2$$

Check:

$$(9(\sqrt{2}))^2 - 43(2) = 76$$
$$9^2\left(\sqrt{2}\right)^2 - 86 = 76$$
$$81(2) - 86 = 76$$
$$162 - 86 = 76$$
$$76 = 76$$

6. **Determine the length of the hypotenuse of the right triangle below.**



$3^2 + 7^2 = x^2$
$9 + 49 = x^2$
$58 = x^2$
$\pm\sqrt{58} = \sqrt{x^2}$
$\pm\sqrt{58} = x$

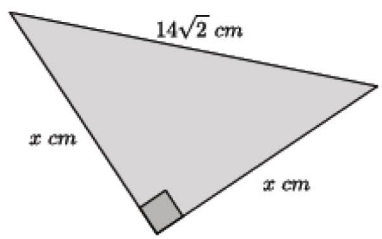
$\sqrt{58} = x$

A negative number would not make sense as a length, so $x = \sqrt{58}$.

Check:

$3^2 + 7^2 = \sqrt{58}^2$
$9 + 49 = 58$
$58 = 58$

7. **Determine the length of the legs in the right triangle below.**



$x^2 + x^2 = \left(14\sqrt{2}\right)^2$
$2x^2 = 14^2\left(\sqrt{2}\right)^2$
$2x^2 = 196(2)$
$\frac{2x^2}{2} = \frac{196(2)}{2}$
$x^2 = 196$
$\sqrt{x^2} = \pm\sqrt{196}$
$x = \pm\sqrt{14^2}$
$x = \pm 14$
$x = 14$

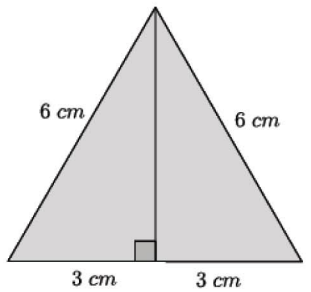
A negative number would not make sense as a length, so $x = 14$.

Check:

$14^2 + 14^2 = \left(14\sqrt{2}\right)^2$
$196 + 196 = 14^2\left(\sqrt{2}\right)^2$
$392 = 196(2)$
$392 = 392$

8. **An equilateral triangle has side lengths of 6 cm. What is the height of the triangle? What is the area of the triangle?**

Note: This problem has two solutions, one with a simplified root and one without. Choose the appropriate solution for your classes based on how much simplifying you have taught them.



Let h represent the height of the triangle.

$$\begin{aligned} 3^2 + h^2 &= 6^2 \\ 9 + h^2 &= 36 \\ 9 - 9 + h^2 &= 36 - 9 \\ h^2 &= 27 \\ \sqrt{h^2} &= \sqrt{27} \\ h &= \sqrt{27} \\ h &= \sqrt{3^3} \\ h &= \sqrt{3^2} \times \sqrt{3} \\ h &= 3\sqrt{3} \end{aligned}$$

Let A represent the area of the triangle.

$$\begin{aligned} A &= \frac{6(3\sqrt{3})}{2} \\ A &= 3(3\sqrt{3}) \\ A &= 9\sqrt{3} \end{aligned}$$

The height of the triangle is $3\sqrt{3}$ cm and the area is $9\sqrt{3}$ cm².

The height of the triangle is $\sqrt{27}$ cm and the area is $3\sqrt{27}$ cm².

9. **Challenge: $\left(\frac{1}{2}x\right)^2 - 3x = 7x + 8 - 10x$. Find the positive value of x that makes the equation true.**

$$\begin{aligned} \left(\frac{1}{2}x\right)^2 - 3x &= 7x + 8 - 10x \\ \frac{1}{4}x^2 - 3x &= -3x + 8 \\ \frac{1}{4}x^2 - 3x + 3x &= -3x + 3x + 8 \\ \frac{1}{4}x^2 &= 8 \\ 4\left(\frac{1}{4}\right)x^2 &= 8(4) \\ x^2 &= 32 \\ \sqrt{x^2} &= \sqrt{32} \\ x &= \sqrt{2^5} \\ x &= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2} \\ x &= 4\sqrt{2} \end{aligned}$$

Check:

$$\begin{aligned} \left(\frac{1}{2}(4\sqrt{2})\right)^2 - 3(4\sqrt{2}) &= 7(4\sqrt{2}) + 8 - 10(4\sqrt{2}) \\ \frac{1}{4}(16)(2) - 3(4\sqrt{2}) &= 7(4\sqrt{2}) - 10(4\sqrt{2}) + 8 \\ \frac{32}{4} - 3(4\sqrt{2}) &= 7(4\sqrt{2}) - 10(4\sqrt{2}) + 8 \\ 8 - 3(4\sqrt{2}) &= (7 - 10)(4\sqrt{2}) + 8 \\ 8 - 3(4\sqrt{2}) &= -3(4\sqrt{2}) + 8 \\ 8 - 8 - 3(4\sqrt{2}) &= -3(4\sqrt{2}) + 8 - 8 \\ -3(4\sqrt{2}) &= -3(4\sqrt{2}) \end{aligned}$$

10. **Challenge: $11x + x(x - 4) = 7(x + 9)$. Find the positive value of x that makes the equation true.**

$$\begin{aligned} 11x + x(x - 4) &= 7(x + 9) \\ 11x + x^2 - 4x &= 7x + 63 \\ 7x + x^2 &= 7x + 63 \\ 7x - 7x + x^2 &= 7x - 7x + 63 \\ x^2 &= 63 \\ \sqrt{x^2} &= \sqrt{63} \\ x &= \sqrt{3^2 \times 7} \\ x &= \sqrt{3^2} \times \sqrt{7} \\ x &= 3\sqrt{7} \end{aligned}$$

Check:

$$\begin{aligned} 11(3\sqrt{7}) + 3\sqrt{7}(3\sqrt{7} - 4) &= 7(3\sqrt{7} + 9) \\ 33\sqrt{7} + 3^2(\sqrt{7})^2 - 4(3\sqrt{7}) &= 21\sqrt{7} + 63 \\ 33\sqrt{7} - 4(3\sqrt{7}) + 9(7) &= 21\sqrt{7} + 63 \\ 33\sqrt{7} - 12\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ (33 - 12)\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ 21\sqrt{7} + 63 &= 21\sqrt{7} + 63 \\ 21\sqrt{7} + 63 - 63 &= 21\sqrt{7} + 63 - 63 \\ 21\sqrt{7} &= 21\sqrt{7} \end{aligned}$$