Name _____ Date_____

Comparison of Numbers Written in Scientific Notation and Interpreting Scientific Notation Using Technology

1. Compare 2.01×10^{15} and 2.8×10^{13}. Which number is larger?

2. The wavelength of the color red is about 6.5×10^{-9} m. The wavelength of the color blue is about 4.75×10^{-9} m. Show that the wavelength of red is longer than the wavelength of blue.

1. Write out a detailed proof of the fact that, given two numbers in scientific notation, $a \times 10^n$ and $b \times 10^n$, $a < b$, if and only if $a \times 10^n < b \times 10^n$.
   a. Let A and B be two positive numbers, with no restrictions on their size. Is it true that $A \times 10^{-5} < B \times 10^5$?
   b. Now, if $A \times 10^{-5}$ and $B \times 10^5$ are written in scientific notation, is it true that $A \times 10^{-5} < B \times 10^5$? Explain.

2. The mass of a neutron is approximately 1.674927×10^{-27} kg. Recall that the mass of a proton is 1.672622×10^{-27} kg. Explain which is heavier.

3. The average lifetime of the Z boson is approximately 3×10^{-25} seconds, and the average lifetime of a neutral rho meson is approximately 4.5×10^{-24} seconds.
   a. Without using the theorem from today's lesson, explain why the neutral rho meson has a longer average lifetime.
   b. Approximately how much longer is the lifetime of a neutral rho meson than a Z boson?

1. **Compare 2.01×10^{15} and 2.8×10^{13}. Which number is larger?**

   $2.01 \times 10^{15} = 2.01 \times 10^{2} \times 10^{13} = 201 \times 10^{13}$

   Since $201 > 2.8$, we have $201 \times 10^{13} > 2.8 \times 10^{13}$, and since $201 \times 10^{13} = 2.01 \times 10^{15}$, we conclude $2.01 \times 10^{15} > 2.8 \times 10^{13}$.

2. **The wavelength of the color red is about 6.5×10^{-9} m. The wavelength of the color blue is about 4.75×10^{-9} m. Show that the wavelength of red is longer than the wavelength of blue.**

   We only need to compare 6.5 and 4.75:

   $6.5 \times 10^{-9} = 650 \times 10^{-7}$ and $4.75 \times 10^{-9} = 475 \times 10^{-7}$, so we see that $650 > 475$.
   Therefore, $6.5 \times 10^{-9} > 4.75 \times 10^{-9}$.

1. **Write out a detailed proof of the fact that, given two numbers in scientific notation, $a \times 10^{n}$ and $b \times 10^{n}$, $a < b$, if and only if $a \times 10^{n} < b \times 10^{n}$.**

   Because $10^{n} > 0$, we can use inequality (A) (i.e., (1) above) twice to draw the necessary conclusions. First, if $a < b$, then by inequality (A), $a \times 10^{n} < b \times 10^{n}$. Second, given $a \times 10^{n} < b \times 10^{n}$, we can use inequality (A) again to show $a < b$ by multiplying each side of $a \times 10^{n} < b \times 10^{n}$ by 10^{-n}.

   a. **Let A and B be two positive numbers, with no restrictions on their size. Is it true that $A \times 10^{-5} < B \times 10^{5}$?**

      No, it is not true that $A \times 10^{-5} < B \times 10^{5}$. Using inequality (A), we can write $A \times 10^{-5} \times 10^{5} < B \times 10^{5} \times 10^{5}$, which is the same as $A < B \times 10^{10}$. To disprove the statement, all we would need to do is find a value of A that exceeds $B \times 10^{10}$.

   b. **Now, if $A \times 10^{-5}$ and $B \times 10^{5}$ are written in scientific notation, is it true that $A \times 10^{-5} < B \times 10^{5}$? Explain.**

      Yes, since the numbers are written in scientific notation, we know that the restrictions for A and B are $1 \leq A < 10$ and $1 \leq B < 10$. The maximum value for A, when multiplied by 10^{-5}, will still be less than 1. The minimum value of B will produce a number at least 10^{5} in size.

2. **The mass of a neutron is approximately 1.674927×10^{-27} kg. Recall that the mass of a proton is 1.672622×10^{-27} kg. Explain which is heavier.**

   Since both numbers have a factor of 10^{-27}, we only need to look at 1.674927 and 1.672622. When we multiply each number by 10^{6}, we get

   1.674927×10^{6} and 1.672622×10^{6},

   which is the same as

   $1,674,927$ and $1,672,622$.

   Now that we are looking at whole numbers, we can see that $1,674,927 > 1,672,622$ (by (2b) above), which means that $1.674927 \times 10^{-27} > 1.672622 \times 10^{-27}$. Therefore, the mass of a neutron is heavier.

3. The average lifetime of the Z boson is approximately 3×10^{-25} seconds, and the average lifetime of a neutral rho meson is approximately 4.5×10^{-24} seconds.

   a. Without using the theorem from today's lesson, explain why the neutral rho meson has a longer average lifetime.

   Since $3 \times 10^{-25} = 3 \times 10^{-1} \times 10^{-24}$, we can compare $3 \times 10^{-1} \times 10^{-24}$ and 4.5×10^{-24}. Based on Example 3 or by use of (1) above, we only need to compare 3×10^{-1} and 4.5, which is the same as 0.3 and 4.5. If we multiply each number by 10, we get whole numbers 3 and 45. Since $3 < 45$, then $3 \times 10^{-25} < 4.5 \times 10^{-24}$. Therefore, the neutral rho meson has a longer average lifetime.

   b. Approximately how much longer is the lifetime of a neutral rho meson than a Z boson?

   $45:3$ *or* 15 *times longer.*