Name _____ Date _____

Nature of Solutions of a System of Linear Equations

Determine the nature of the solution to each system of linear equations. If the system has a solution, then find it without graphing.

1. $\begin{cases} y = \frac{1}{2}x + \frac{5}{2} \\ x - 2y = 7 \end{cases}$

2. $\begin{cases} y = \frac{2}{3}x + 4 \\ 2y + \frac{1}{2}x = 2 \end{cases}$

3. $\begin{cases} y = 3x - 2 \\ -3x + y = -2 \end{cases}$

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1. $\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$

2. $\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$

3. $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$

4. $\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$

5. $\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$

6. $\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$

7. $\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$

8. $\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$

9. $\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$

10. $\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$

11. Write a system of linear equations with $(-3, 9)$ as its solution.

Determine the nature of the solution to each system of linear equations. If the system has a solution, then find it without graphing.

1. $\begin{cases} y = \frac{1}{2}x + \frac{5}{2} \\ x - 2y = 7 \end{cases}$

The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.

2. $\begin{cases} y = \frac{2}{3}x + 4 \\ 2y + \frac{1}{2}x = 2 \end{cases}$

The slopes of these two equations are unique. That means they graph as distinct lines and will intersect at one point. Therefore, this system has one solution.

$$2\left(\frac{2}{3}x + 4\right) + \frac{1}{2}x = 2$$
$$\frac{4}{3}x + 8 + \frac{1}{2}x = 2$$
$$\frac{11}{6}x + 8 = 2$$
$$\frac{11}{6}x = -6$$
$$x = -\frac{36}{11}$$

$$y = \frac{2}{3}\left(-\frac{36}{11}\right) + 4$$
$$y = -\frac{24}{11} + 4$$
$$y = \frac{20}{11}$$

The solution is $\left(-\frac{36}{11}, \frac{20}{11}\right)$.

3. $\begin{cases} y = 3x - 2 \\ -3x + y = -2 \end{cases}$

These equations define the same line. Therefore, this system will have infinitely many solutions.

Students practice determining the nature of solutions of a system of linear equations and finding the solution for systems that have one.

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1. $\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$

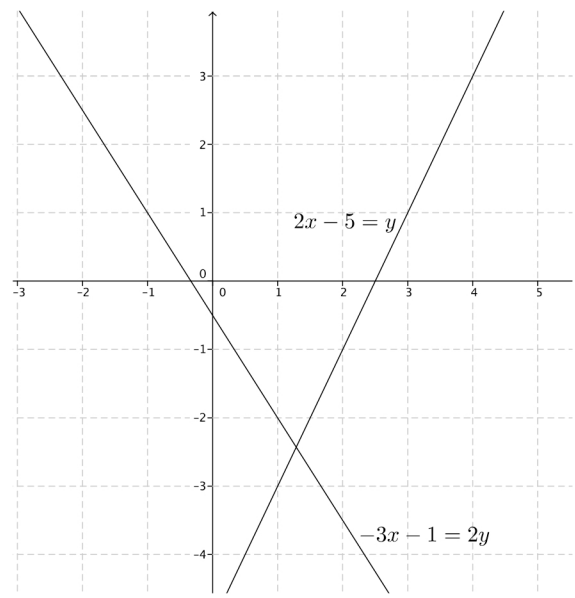
The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.

2. $\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$

$$(2x - 5 = y)2$$
$$4x - 10 = 2y$$
$$\begin{cases} 4x - 10 = 2y \\ -3x - 1 = 2y \end{cases}$$
$$4x - 10 = -3x - 1$$
$$7x - 10 = -1$$
$$7x = 9$$
$$x = \frac{9}{7}$$

$$y = 2\left(\frac{9}{7}\right) - 5$$
$$y = \frac{18}{7} - 5$$
$$y = -\frac{17}{7}$$

The solution is $\left(\frac{9}{7}, -\frac{17}{7}\right)$.

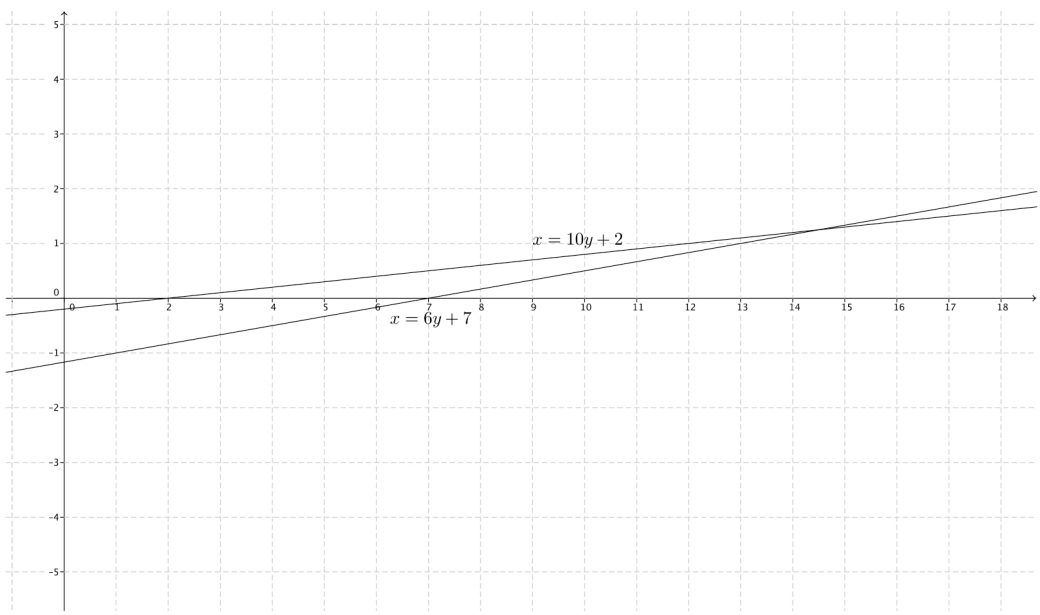


3. $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$

$$\begin{aligned} 6y + 7 &= 10y + 2 \\ 7 &= 4y + 2 \\ 5 &= 4y \\ \frac{5}{4} &= y \end{aligned}$$

$$\begin{aligned} x &= 6\left(\frac{5}{4}\right) + 7 \\ x &= \frac{15}{2} + 7 \\ x &= \frac{29}{2} \end{aligned}$$

The solution is $\left(\frac{29}{2}, \frac{5}{4}\right)$.



4. $\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$

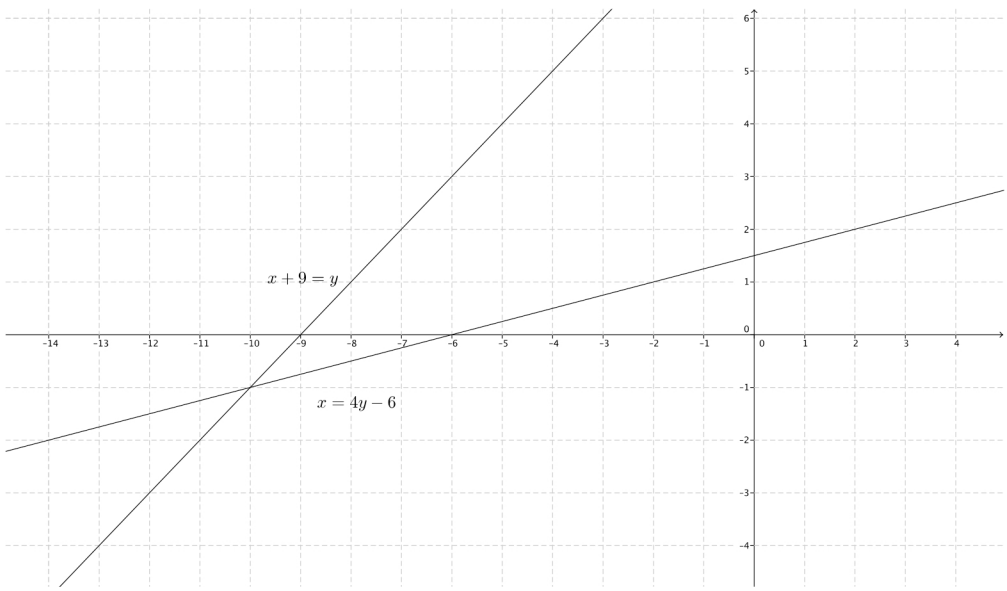
These equations define the same line. Therefore, this system will have infinitely many solutions.

5. $\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$

$$4y - 6 + 9 = y$$
$$4y + 3 = y$$
$$3 = -3y$$
$$-1 = y$$

$$x + 9 = -1$$
$$x = -10$$

The solution is $(-10, -1)$.

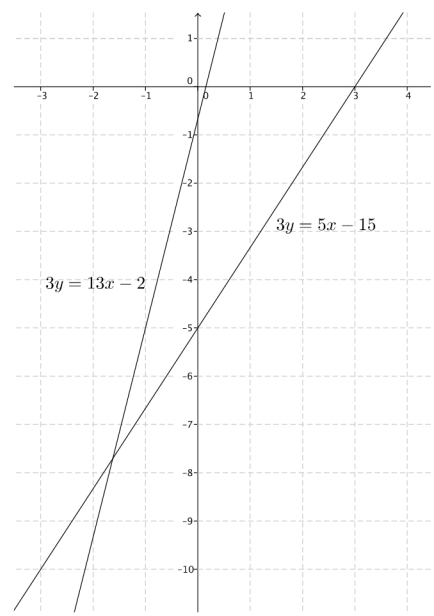


6. $\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$

$$5x - 15 = 13x - 2$$
$$-15 = 8x - 2$$
$$-13 = 8x$$
$$-\frac{13}{8} = x$$

$$3y = 5\left(-\frac{13}{8}\right) - 15$$
$$3y = -\frac{65}{8} - 15$$
$$3y = -\frac{185}{8}$$
$$y = -\frac{185}{24}$$

The solution is $\left(-\frac{13}{8}, -\frac{185}{24}\right)$.



7. $\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$

These equations define the same line. Therefore, this system will have infinitely many solutions.

8. $\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$

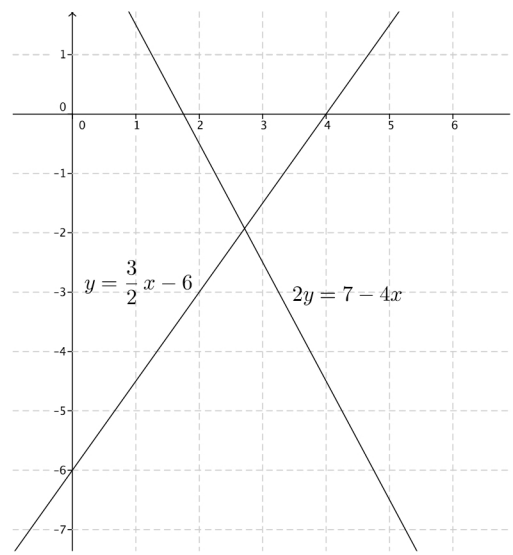
The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.

9. $\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$

$$2\left(y = \frac{3}{2}x - 6\right)$$
$$2y = 3x - 12$$
$$\begin{cases} 2y = 3x - 12 \\ 2y = 7 - 4x \end{cases}$$
$$3x - 12 = 7 - 4x$$
$$7x - 12 = 7$$
$$7x = 19$$
$$x = \frac{19}{7}$$

$$y = \frac{3}{2}\left(\frac{19}{7}\right) - 6$$
$$y = \frac{57}{14} - 6$$
$$y = -\frac{27}{14}$$

The solution is $\left(\frac{19}{7}, -\frac{27}{14}\right)$.

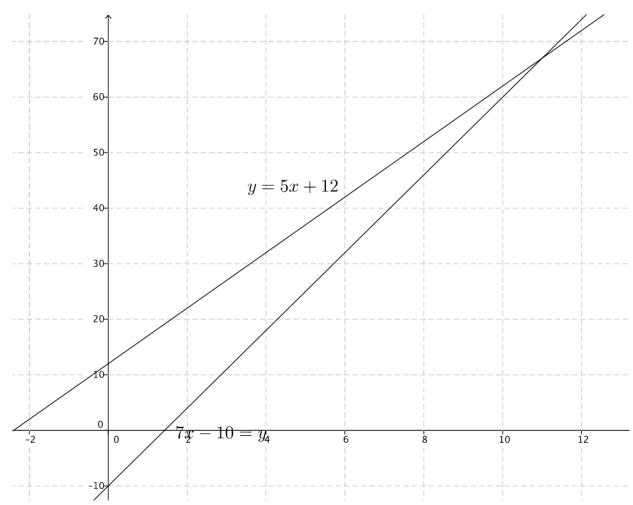


10. $\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$

$$7x - 10 = 5x + 12$$
$$2x - 10 = 12$$
$$2x = 22$$
$$x = 11$$

$$y = 5(11) + 12$$
$$y = 55 + 12$$
$$y = 67$$

The solution is $(11, 67)$.



11. **Write a system of linear equations with $(-3, 9)$ as its solution.**

Answers will vary. Verify that students have written a system of equations where $(-3, 9)$ is a solution to each equation in the system. Sample solution: $\begin{cases} y = x + 12 \\ x + y = 6 \end{cases}$