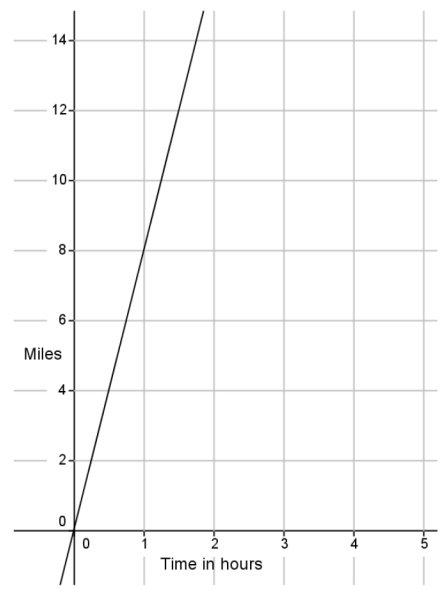
Name _____ Date _____

Comparing Linear Functions and Graphs

Brothers, Paul and Pete, walk 2 miles to school from home. Paul can walk to school in 24 minutes. Pete has slept in again and needs to run to school. Paul walks at constant rate, and Pete runs at a constant rate. The graph of the function that represents Pete's run is shown below.

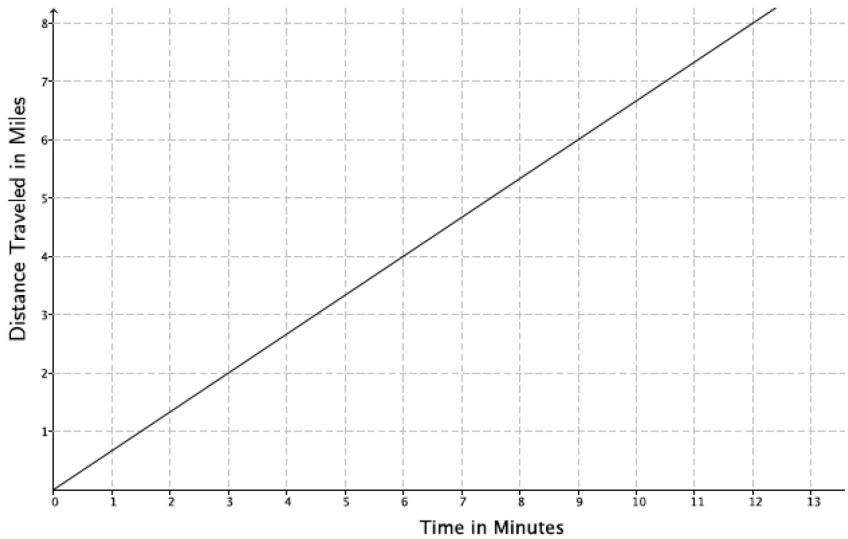
a. Which brother is moving at a greater rate? Explain how you know.



b. If Pete leaves 5 minutes after Paul, will he catch Paul before they get to school?

1. The graph below represents the distance, y, Car A travels in x minutes. The table represents the distance, y, Car B travels in x minutes. Which car is traveling at a greater speed? How do you know?

   Car A:



   Car B:

Time in minutes (x)	Distance (y)
15	12.5
30	25
45	37.5

2. The local park needs to replace an existing fence that is 6 feet high. Fence Company A charges \$7,000 for building materials and \$200 per foot for the length of the fence. Fence Company B charges based on the length of the fence. That is, the total cost of the six-foot high fence will depend on how long the fence is. The table below represents the inputs and the corresponding outputs that the function assigns for Fence Company B.

Input (length of fence)	Output (cost of bill)
100	\$26,000
120	\$31,200
180	\$46,800
250	\$65,000

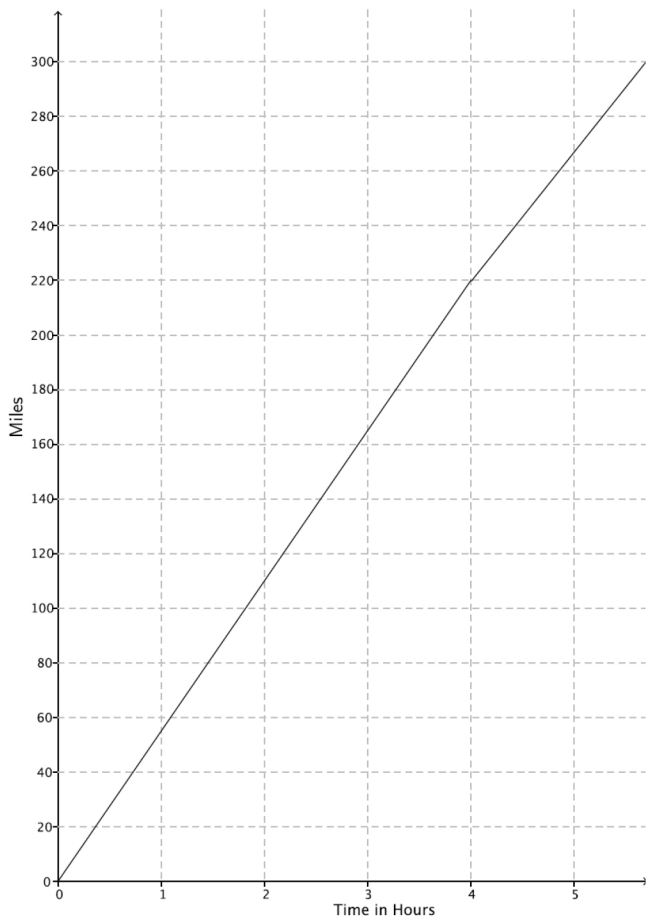
   a. Which company charges a higher rate per foot of fencing? How do you know?

   b. At what number of the length of the fence would the cost from each fence company be the same? What will the cost be when the companies charge the same amount? If the fence you need is 190 feet in length, which company would be a better choice?

3. The rule $y = 123x$ is used to describe the function for the number of minutes needed, x, to produce y toys at Toys Plus. Another company, #1 Toys, has a similar function that assigned the values shown in the table below. Which company produces toys at a slower rate? Explain.

Time in minutes (x)	Toys Produced (y)
5	600
11	1,320
13	1,560

4. A function describes the number of miles a train can travel, y, for the number of hours, x. The figure shows the graph of this function. Assume that the train travels at a constant speed. The train is traveling from City A to City B (a distance of 320 miles). After 4 hours, the train slows down to a constant speed of 48 miles per hour.



a. How long will it take the train to reach its destination?

b. If the train had not slowed down after 4 hours, how long would it have taken to reach its destination?

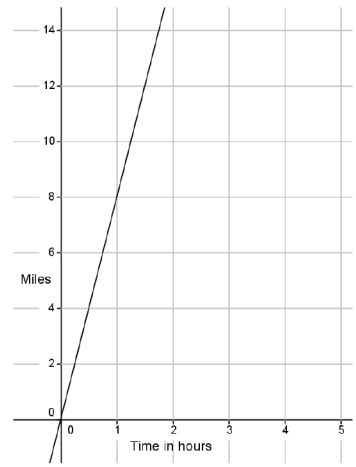
c. Suppose after 4 hours, the train increased its constant speed. How fast would the train have to travel to complete the destination in 1.5 hours?

d. A hose is used to fill up a $1{,}200$ gallon water truck at a constant rate. After 10 minutes, there are 65 gallons of water in the truck. After 15 minutes, there are 82 gallons of water in the truck. How long will it take to fill up the water truck?

e. The driver of the truck realizes that something is wrong with the hose he is using. After 30 minutes, he shuts off the hose and tries a different hose. The second hose has a constant rate of 18 gallons per minute. How long does it take the second hose to fill up the truck?

f. Could there ever be a time when the first hose and the second hose filled up the same amount of water?

Brothers, Paul and Pete, walk 2 miles to school from home. Paul can walk to school in 24 minutes. Pete has slept in again and needs to run to school. Paul walks at constant rate, and Pete runs at a constant rate. The graph of the function that represents Pete's run is shown below.



a. **Which brother is moving at a greater rate? Explain how you know.**

Paul takes 24 minutes to walk 2 miles; therefore, his rate is $\frac{1}{12}$.

Pete can run 8 miles in 60 minutes; therefore, his rate is $\frac{8}{60}$, *or* $\frac{2}{15}$.

Since $\frac{2}{15} > \frac{1}{12}$, *Pete is moving at a greater rate.*

b. **If Pete leaves 5 minutes after Paul, will he catch Paul before they get to school?**

Student solution methods will vary. Sample answer is shown.

Since Pete slept in, we need to account for that fact. So, Pete's time would be decreased. The equation that would represent the number of miles Pete walks, y, *walked in* x *minutes, would be* $y = \frac{2}{15}(x - 5)$.

The equation that would represent the number of miles Paul runs, y, *run in* x *minutes, would be* $y = \frac{1}{12}x$.

To find out when they meet, solve the system of equations:

$$\begin{cases} y = \frac{2}{15}x - \frac{2}{3} \\ y = \frac{1}{12}x \end{cases}$$

$$\frac{2}{15}x - \frac{2}{3} = \frac{1}{12}x$$

$$\frac{2}{15}x - \frac{2}{3} - \frac{1}{12}x + \frac{2}{3} = \frac{1}{12}x - \frac{1}{12}x + \frac{2}{3}$$

$$\frac{1}{20}x = \frac{2}{3}$$

$$\left(\frac{20}{1}\right)\frac{1}{20}x = \frac{2}{3}\left(\frac{20}{1}\right)$$

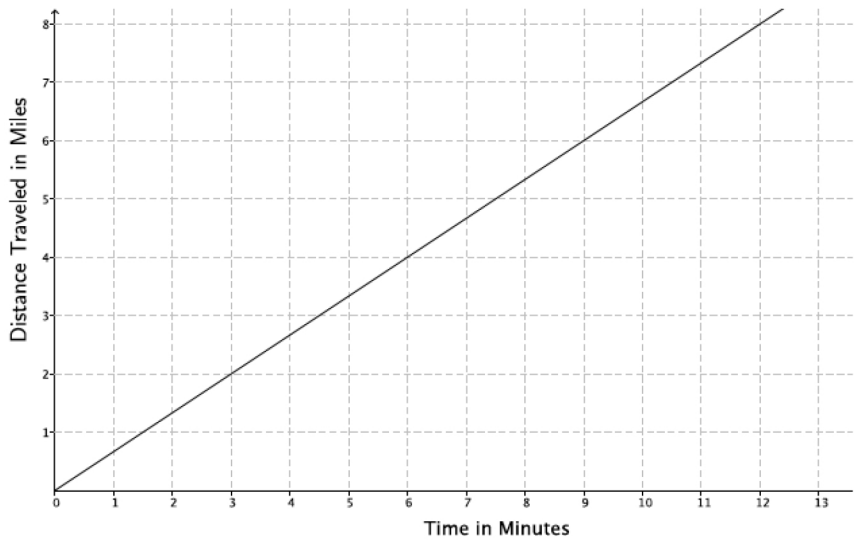
$$x = x\frac{40}{3}$$

$$y = \left(\frac{40}{3}\right) = \frac{10}{9} \quad \textit{or} \quad y = \frac{2}{15}\left(\frac{40}{3}\right) - \frac{2}{3}$$

Pete would catch up to Paul in $\frac{40}{9}$ *minutes, which is equal to* $\frac{10}{9}$ *miles. Yes, he will catch Paul before they get to school because it is less than the total distance, two miles, to school.*

1. **The graph below represents the distance, y, Car A travels in x minutes. The table represents the distance, y, Car B travels in x minutes. Which car is traveling at a greater speed? How do you know?**

Car A:



Car B:

Time in minutes (x)	Distance (y)
15	12.5
30	25
45	37.5

Based on the graph, Car A is traveling at a rate of 2 miles every 3 minutes, $m = \frac{2}{3}$. From the table, the rate that Car B is traveling is constant, as shown below.

$$\frac{25 - 12.5}{30 - 15} = \frac{12.5}{15} = \frac{25}{30} = \frac{5}{6}$$
$$\frac{37.5 - 25}{45 - 30} = \frac{12.5}{15} = \frac{5}{6}$$
$$\frac{37.5 - 12.5}{45 - 15} = \frac{25}{30} = \frac{5}{6}$$

Since $\frac{5}{6} > \frac{2}{3}$, Car B is traveling at a greater speed.

2. The local park needs to replace an existing fence that is 6 feet high. Fence Company A charges $\$7,000$ for building materials and $\$200$ per foot for the length of the fence. Fence Company B charges based on the length of the fence. That is, the total cost of the 6-foot high fence will depend on how long the fence is. The table below represents the inputs and the corresponding outputs that the function for Fence Company B assigns.

Input (length of fence)	Output (cost of bill)
100	$\$26,000$
120	$\$31,200$
180	$\$46,800$
250	$\$65,000$

a. Which company charges a higher rate per foot of fencing? How do you know?

Let x represent the length of the fence and y represent the total cost.

The equation that represents the function for Fence Company A is $y = 200x + 7,000$. So, the rate is 200.

The rate of change for Fence Company B:

$$\frac{26,000 - 31,200}{100 - 120} = \frac{-5,200}{-20} = 260 \qquad \frac{31,200 - 46,800}{120 - 180} = \frac{-15,600}{-60} = 260 \qquad \frac{46,800 - 65,000}{180 - 250} = \frac{-18,200}{-70} = 260$$

Fence Company B charges a higher rate per foot because when you compare the rates, $260 > 200$.

b. At what number of the length of the fence would the cost from each fence company be the same? What will the cost be when the companies charge the same amount? If the fence you need is 190 feet in length, which company would be a better choice?

Student solutions will vary. Sample solution is provided.

The equation for Fence Company B is

$$y = 260x.$$

We can find out at what point the fence companies charge the same amount by solving the system:

$$\begin{cases} y = 200x + 7000 \\ y = 260x \end{cases}$$

$$\begin{aligned} 200x + 7,000 &= 260x \\ 7,000 &= 60x \\ 116.6666\ldots &= x \\ 116.6 &\approx x \end{aligned}$$

At 116.6 feet of fencing, both companies would charge the same amount (about $\$30,320$). Less than 116.6 feet of fencing means that the cost from Fence Company A will be more than Fence Company B. More than 116.6 feet of fencing means that the cost from Fence Company B will be more than Fence Company A. So, for 190 feet of fencing, Fence Company A is the better choice.

3. The rule $y = 123x$ is used to describe the function for the number of minutes needed, x, to produce y toys at Toys Plus. Another company, #1 Toys, has a similar function that assigned the values shown in the table below. Which company produces toys at a slower rate? Explain.

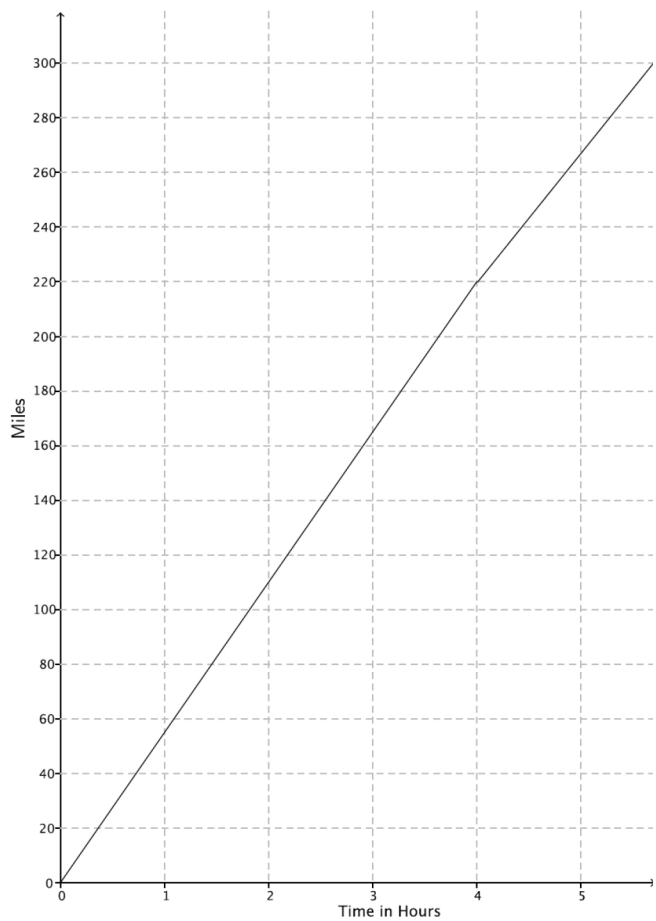
Time in minutes (x)	Toys Produced (y)
5	600
11	$1,320$
13	$1,560$

#1 Toys produces toys at a constant rate because the data in the table increases at a constant rate, as shown below.

$$\frac{1,320 - 600}{11 - 5} = \frac{720}{6} = 120 \qquad \frac{1,560 - 600}{13 - 5} = \frac{960}{8} = 120 \qquad \frac{1,560 - 1,320}{13 - 11} = \frac{240}{2} = 120$$

The rate of production for Toys Plus is 123 and for #1 Toys is 120. Since $120 < 123$, #1 Toys produces toys at a slower rate.

4. A function describes the number of miles a train can travel, y, for the number of hours, x. The figure shows the graph of this function. Assume that the train travels at a constant speed. The train is traveling from City A to City B (a distance of 320 miles). After 4 hours, the train slows down to a constant speed of 48 miles per hour.



a. **How long will it take the train to reach its destination?**

Student solutions will vary. Sample solution is provided.

The equation for the graph is $y = 55x$. If the train travels for 4 hours at a rate of 55 miles per hour, it will have travelled 220 miles. That means it has 100 miles to get to its destination. The equation for the second part of the journey is $y = 48x$. Then,

$$100 = 48x$$
$$2.08333\ldots. = x$$
$$2 \approx x.$$

This means it will take about 6 hours $(4 + 2 = 6)$ for the train to reach its destination.

b. **If the train had not slowed down after 4 hours, how long would it have taken to reach its destination?**

$$320 = 55x$$
$$5.8181818\ldots. = x$$
$$5.8 \approx x$$

The train would have reached its destination in about 5.8 hours had it not slowed down.

c. **Suppose after 4 hours, the train increased its constant speed. How fast would the train have to travel to complete the destination in 1.5 hours?**

Let m represent the new constant speed of the train; then,

$$100 = m(1.5)$$
$$66.6666\ldots. = x$$
$$66.6 \approx x.$$

The train would have to increase its speed to about 66.6 miles per hour to arrive at its destination 1.5 hours later.

5.

a. **A hose is used to fill up a $1,200$ gallon water truck at a constant rate. After 10 minutes, there are 65 gallons of water in the truck. After 15 minutes, there are 82 gallons of water in the truck. How long will it take to fill up the water truck?**

Student solutions will vary. Sample solution is provided.

Let x represent the time in minutes it takes to pump y gallons of water. Then, the rate can be found as follows:

Time in minutes (x)	*Amount of water pumped in gallons (y)*
10	65
15	82

$$\frac{65-82}{10-15}=\frac{-17}{-5}$$
$$=\frac{17}{5}$$

Since the water is pumping at a constant rate, we can assume the equation is linear. Therefore, the equation for the first hose is found by

$$\begin{cases}10a+b=65\\15a+b=82\end{cases}.$$

If we multiply the first equation by -1:

$$\begin{cases}-10a-b=-65\\15a+b=82\end{cases}$$

$$-10a-b+15a+b=-65+82$$
$$5a=17$$
$$a=\frac{17}{5}$$

$$10\left(\frac{17}{5}\right)+b=65$$
$$b=31$$

The equation for the first hose is $y=\frac{17}{5}x+31$. If the hose needs to pump $1,200$ gallons of water into the truck, then

$$1200=\frac{17}{5}x+31$$
$$1169=\frac{17}{5}x$$
$$343.8235\ldots.=x$$
$$343.8\approx x.$$

It would take about 344 minutes or about 5.7 hours to fill up the truck.

b. **The driver of the truck realizes that something is wrong with the hose he is using. After 30 minutes, he shuts off the hose and tries a different hose. The second hose has a constant rate of 18 gallons per minute. How long does it take the second hose to fill up the truck?**

Since the first hose has been pumping for 30 minutes, there are 133 gallons of water already in the truck. That means the new hose only has to fill up $1,067$ gallons. Since the second hose fills up the truck at a constant rate of 18 gallons per minute, the equation for the second hose is $y = 18x$.

$$1,067 = 18x$$
$$59.27 = x$$

It will take the second hose 59.27 minutes (or a little less than an hour) to finish the job.

c. **Could there ever be a time when the first hose and the second hose filled up the same amount of water?**

To see if the first hose and the second hose could have ever filled up the same amount of water, I would need to solve for the system:

$$\begin{cases} y = 18x \\ y = \frac{17}{5}x + 31 \end{cases}$$
$$18x = \frac{17}{5}x + 31$$
$$\frac{73}{5}x = 31$$
$$x = \frac{155}{73}$$
$$x \approx 2.12$$

The second hose could have filled up the same amount of water as the first hose at about 2 minutes.