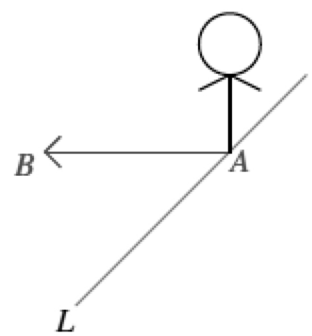
Name ____ Date ____________________

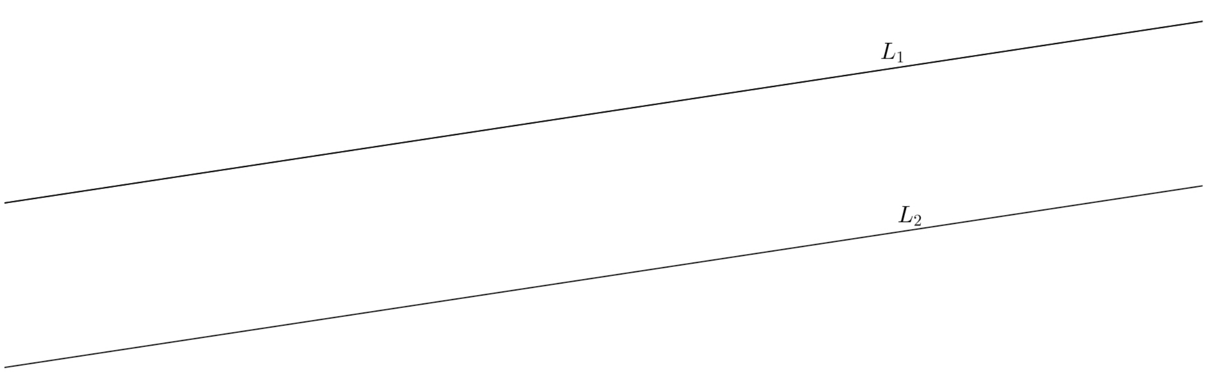
Sequencing Reflections and Translations

Draw a figure, A, a line of reflection, L, and a vector $\overrightarrow{FG}$ in the space below. Show that under a sequence of a translation and a reflection that the sequence of the reflection followed by the translation is not equal to the translation followed by the reflection. Label the figure as A' after finding the location according to the sequence reflection followed by the translation, and label the figure A'' after finding the location according to the composition translation followed by the reflection. If A' is not equal to A'', then we have shown that the sequence of the reflection followed by a translation is not equal to the sequence of the translation followed by the reflection. (This will be proven in high school.)

1. Let there be a reflection across line L, and let there be a translation along vector $\overrightarrow{AB}$, as shown. If S denotes the black figure, compare the translation of S followed by the reflection of S with the reflection of S followed by the translation of S.



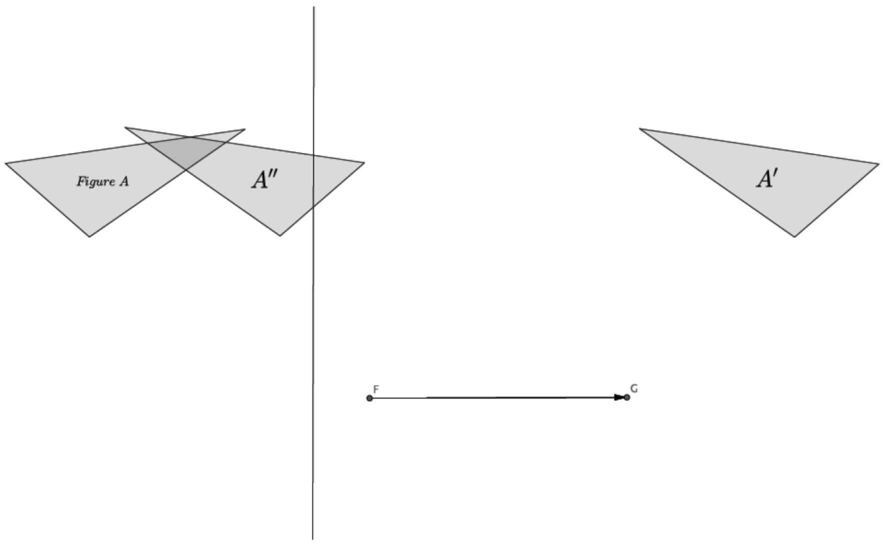
2. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Show that a $Reflection_2$ followed by $Reflection_1$ is not equal to a $Reflection_1$ followed by $Reflection_2$. (Hint: Take a point on L_1 and see what each of the sequences does to it.)



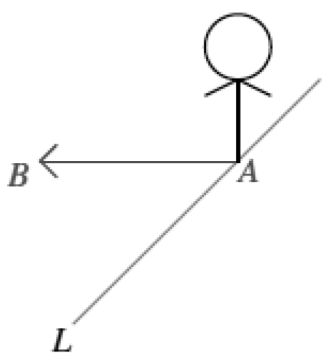
3. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Can you guess what $Reflection_1$ followed by $Reflection_2$ is? Give as persuasive an argument as you can. (Hint: Examine the work you just finished for the last problem.)

Draw a figure, A, a line of reflection, L, and a vector $\overrightarrow{FG}$ in the space below. Show that under a sequence of a translation and a reflection, that the sequence of the reflection followed by the translation is not equal to the translation followed by the reflection. Label the figure as A' after finding the location according to the sequence reflection followed by the translation, and label the figure A''after finding the location according to the composition translation followed by the reflection. If A' is not equal to A'', then we have shown that the sequence of the reflection followed by a translation is not equal to the sequence of the translation followed by the reflection. (This will be proven in high school.)

Sample student drawing:



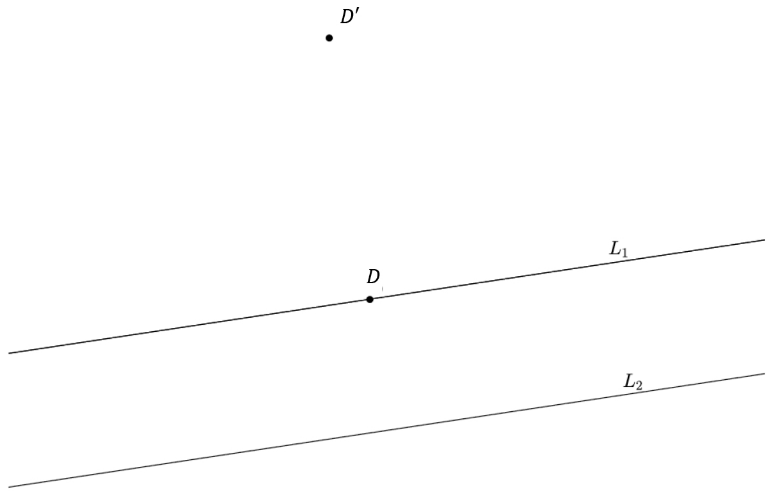
1. Let there be a reflection across line L, and let there be a translation along vector $\overrightarrow{AB}$, as shown. If S denotes the black figure, compare the translation of S followed by the reflection of S with the reflection of S followed by the translation of S.



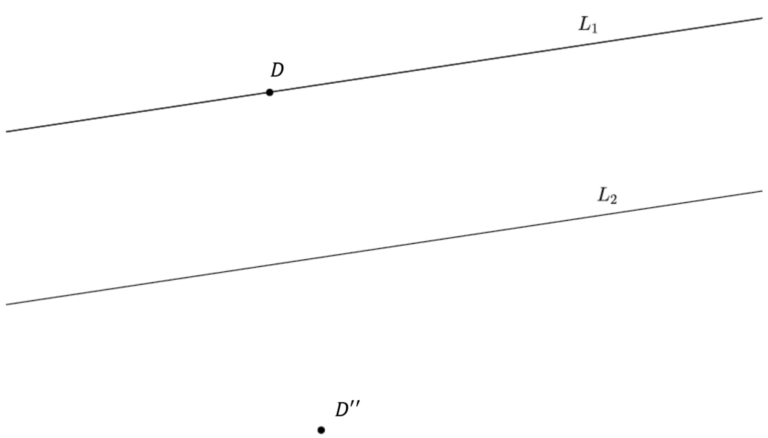
Students should notice that the two sequences place figure S in different locations in the plane.

2. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Show that a $Reflection_2$ followed by $Reflection_1$ is not equal to a $Reflection_1$ followed by $Reflection_2$. (Hint: Take a point on L_1 and see what each of the sequences does to it.)

Let D be a point on L_1, as shown, and let $D' = Reflection_2$ followed by $Reflection_1$. Notice where D' is.



Let $D'' = Reflection_1$ followed by $Reflection_2$. Notice where the D'' is.



Since $D' \neq D''$, the sequences are not equal.

3. **Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Can you guess what $Reflection_1$ followed by $Reflection_2$ is? Give as persuasive an argument as you can. (Hint: Examine the work you just finished for the last problem.)**

The sequence $Reflection_1$ followed by $Reflection_2$ is just like the translation along a vector $\overrightarrow{AB}$, as shown below, where $AB \perp L_1$. The length of AB is equal to twice the distance between L_1 and L_2.