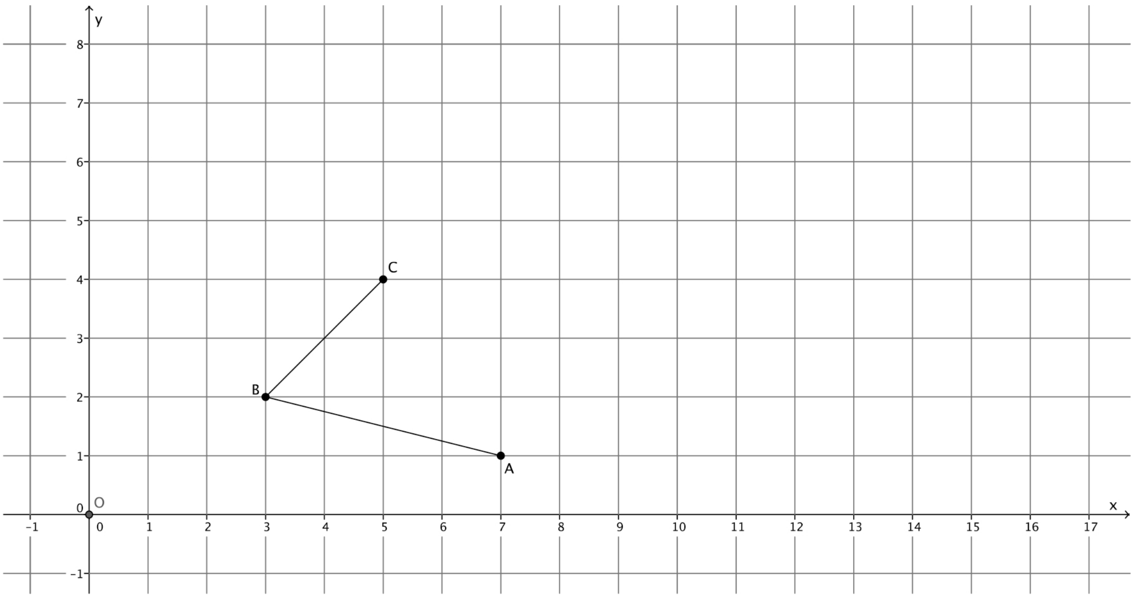
Name ____ Date__________________

Informal Proofs of Properties of Dilations

Dilate $\angle ABC$ with center O and scale factor $r = 2$. Label the dilated angle, $\angle A'B'C'$.



1. If $\angle ABC = 72°$, then what is the measure of $\angle A'B'C'$?

2. If segment AB is 2 cm. What is the measure of segment $A'B'$?

3. Which segments, if any, are parallel?

1. A dilation from center O by scale factor r of a line maps to what? Verify your claim on the coordinate plane.

2. A dilation from center O by scale factor r of a segment maps to what? Verify your claim on the coordinate plane.

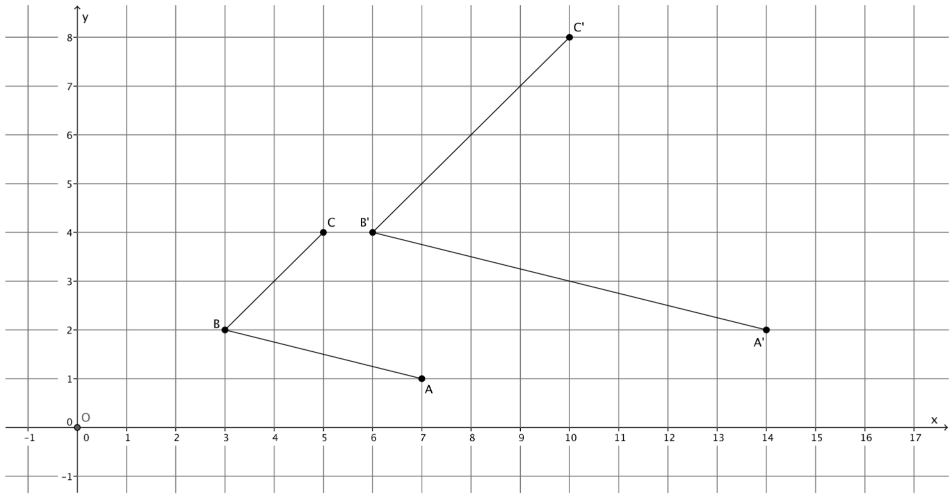
3. A dilation from center O by scale factor r of a ray maps to what? Verify your claim on the coordinate plane.

4. Challenge Problem:

   Prove the theorem: *A dilation maps lines to lines.*

   Let there be a dilation from center O with scale factor r so that $P' = Dilation(P)$ and $Q' = Dilation(Q)$. Show that line PQ maps to line $P'Q'$ (i.e., that dilations map lines to lines). Draw a diagram, and then write your informal proof of the theorem. (Hint: This proof is a lot like the proof for segments. This time, let U be a point on line PQ, that is not between points P and Q.)

Dilate $\angle ABC$ with center O and scale factor $r = 2$. Label the dilated angle, $\angle A'B'C'$.



1. **If $\angle ABC = 72°$, then what is the measure of $\angle A'B'C'$?**

   Since dilations preserve angles, then $\angle A'B'C' = 72°$.

2. **If segment AB is 2 cm. What is the measure of segment $A'B'$?**

   The length of segment $A'B'$ is 4 cm.

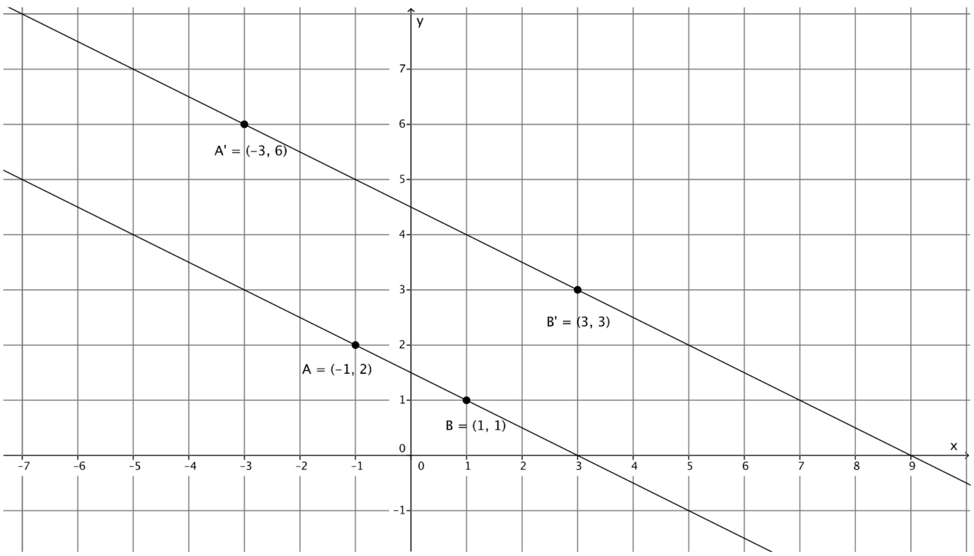
3. **Which segments, if any, are parallel?**

   Since dilations map segments to parallel segments, then $AB \parallel A'B'$, and $BC \parallel B'C'$.

1. **A dilation from center O by scale factor r of a line maps to what? Verify your claim on the coordinate plane.**

   The dilation of a line maps to a line.

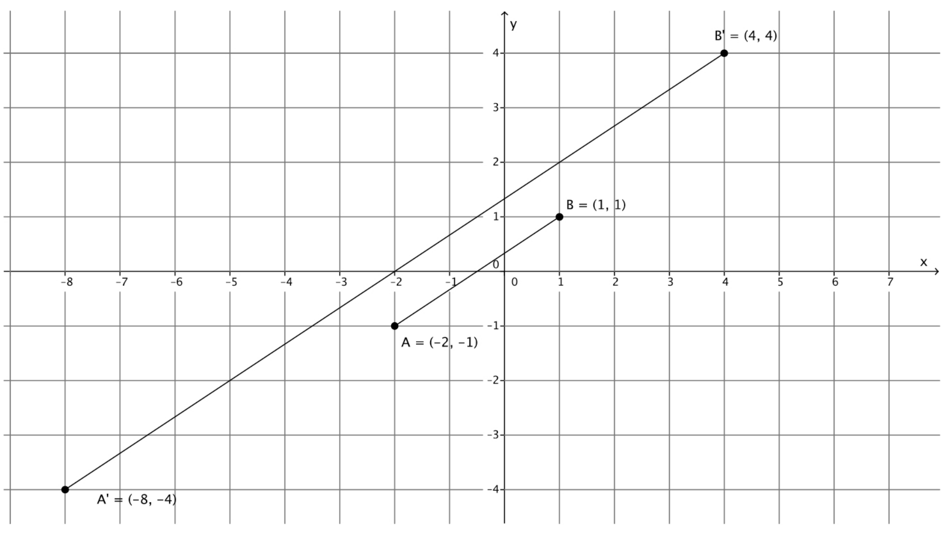
   Sample student work shown below.



2. **A dilation from center O by scale factor r of a segment maps to what? Verify your claim on the coordinate plane.**

   The dilation of a segment maps to a segment.

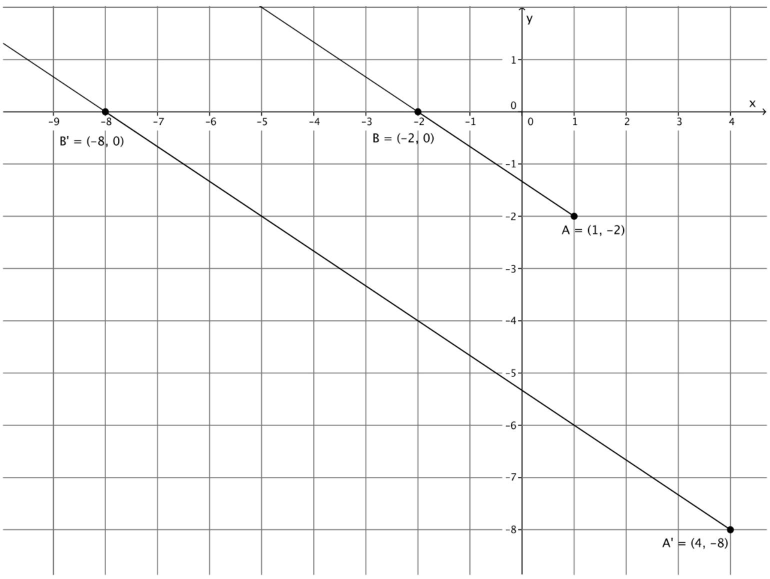
   Sample student work shown below.



3. **A dilation from center O by scale factor r of a ray maps to what? Verify your claim on the coordinate plane.**

The dilation of a ray maps to a ray.

Sample student work shown below.

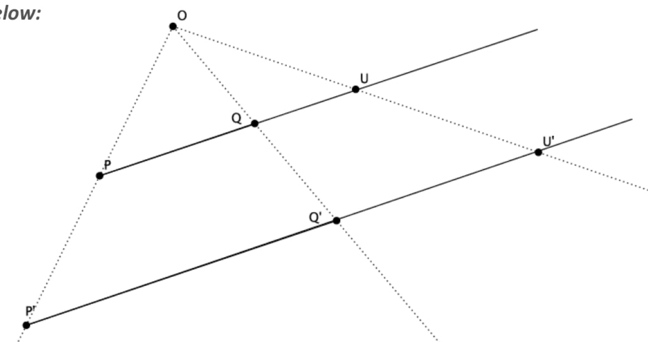


4. **Challenge Problem:**

Prove the theorem: *A dilation maps lines to lines*.

Let there be a dilation from center O with scale factor r so that $P' = Dilation(P)$ and $Q' = Dilation(Q)$. Show that line PQ maps to line $P'Q'$ (i.e., that dilations map lines to lines). Draw a diagram, and then write your informal proof of the theorem. (Hint: This proof is a lot like the proof for segments. This time, let U be a point on line PQ, that is not between points P and Q.)

Sample student drawing and response below:



Let U be a point on line PQ. By definition of dilation, we also know that $U' = Dilation(U)$. We need to show that U' is a point on line $P'Q'$. If we can, then we have proven that a dilation maps lines to lines.

By definition of dilation and FTS, we know that $\frac{|OP'|}{|OP|} = \frac{|OQ'|}{|OQ|}$ and that line PQ is parallel to $P'Q'$. Similarly, we know that $\frac{|OQ'|}{|OQ|} = \frac{|OU'|}{|OU|} = r$ and that line QU is parallel to line $Q'U'$. Since U is a point on line PQ, then we also know that line PQ is parallel to line $Q'U'$. But we already know that PQ is parallel to $P'Q'$. Since there can only be one line that passes through Q' that is parallel to line PQ, then line $P'Q'$ and line $Q'U'$ must coincide. That places the dilation of point U, U', on the line $P'Q'$, which proves that dilations map lines to lines.