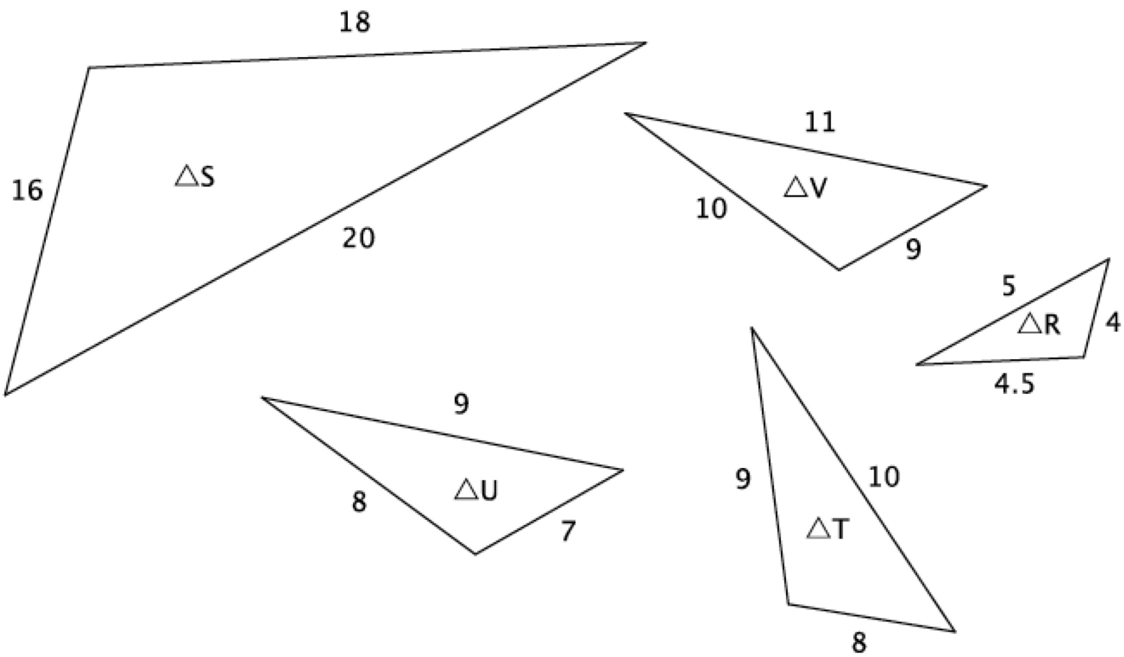
Name ____ Date__________________

Basic Properties of Similarity

Use the diagram below to answer Questions 1 and 2.

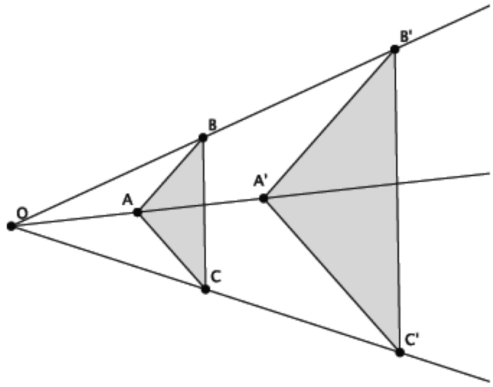


1. Which two triangles, if any, have similarity that is symmetric?

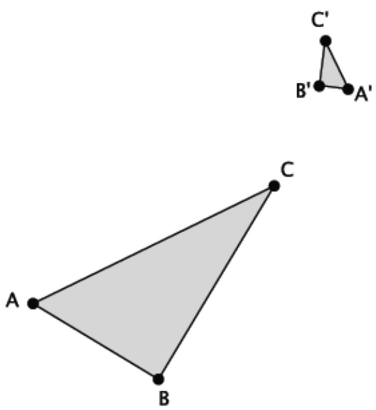
2. Which three triangles, if any, have similarity that is transitive?

1. Would a dilation alone be enough to show that similarity is symmetric? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$? Consider the two examples below.

   a. Given $\triangle ABC \sim \triangle A'B'C'$. Is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.



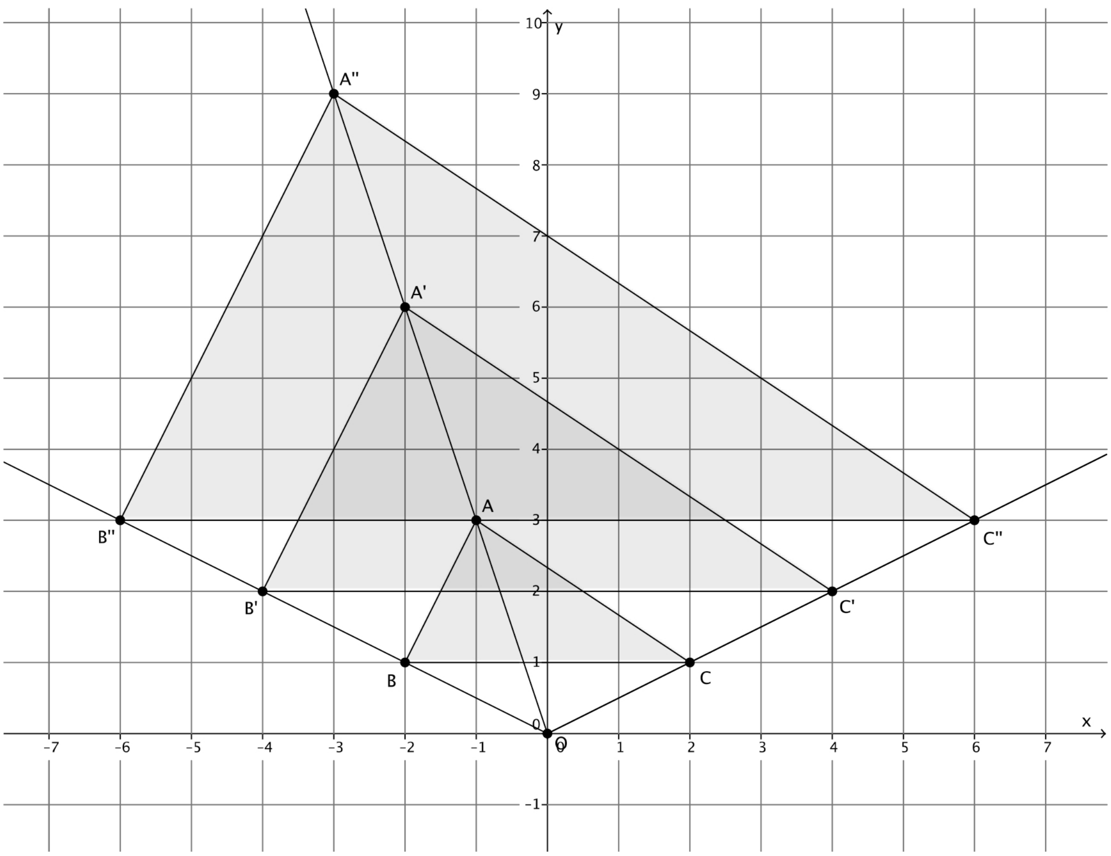
   b. Given $\triangle ABC \sim \triangle A'B'C'$. Is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.



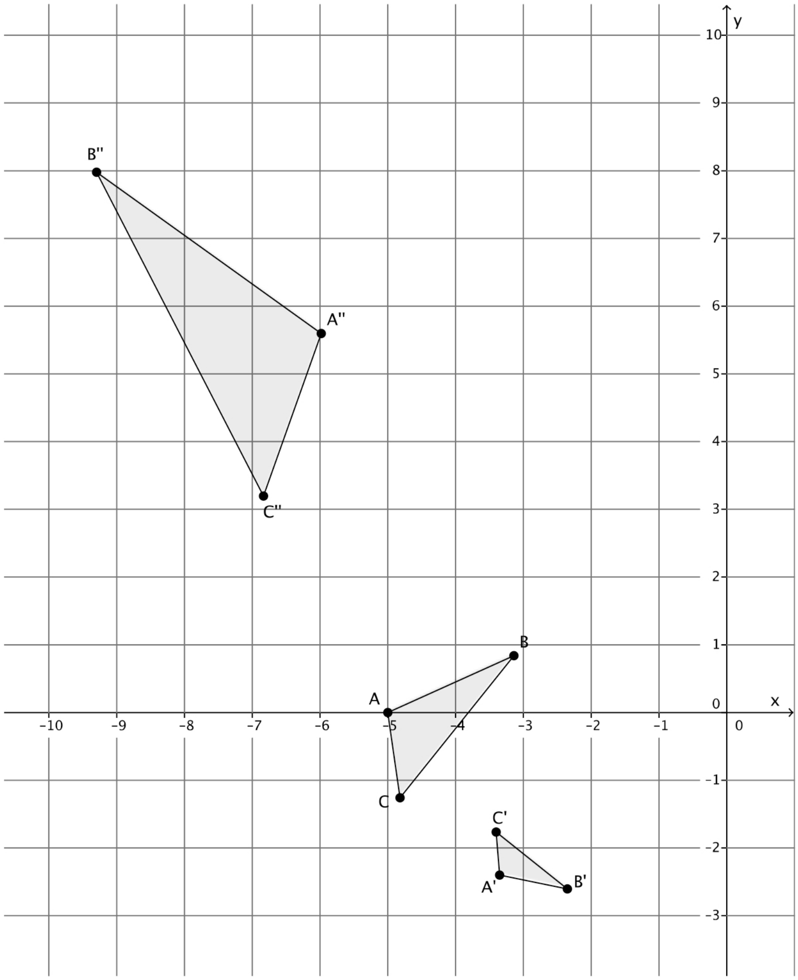
   c. In general, is dilation enough to prove that similarity is a symmetric relation? Explain.

2. Would a dilation alone be enough to show that similarity is transitive? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$, and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$? Consider the two examples below.

a. Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.

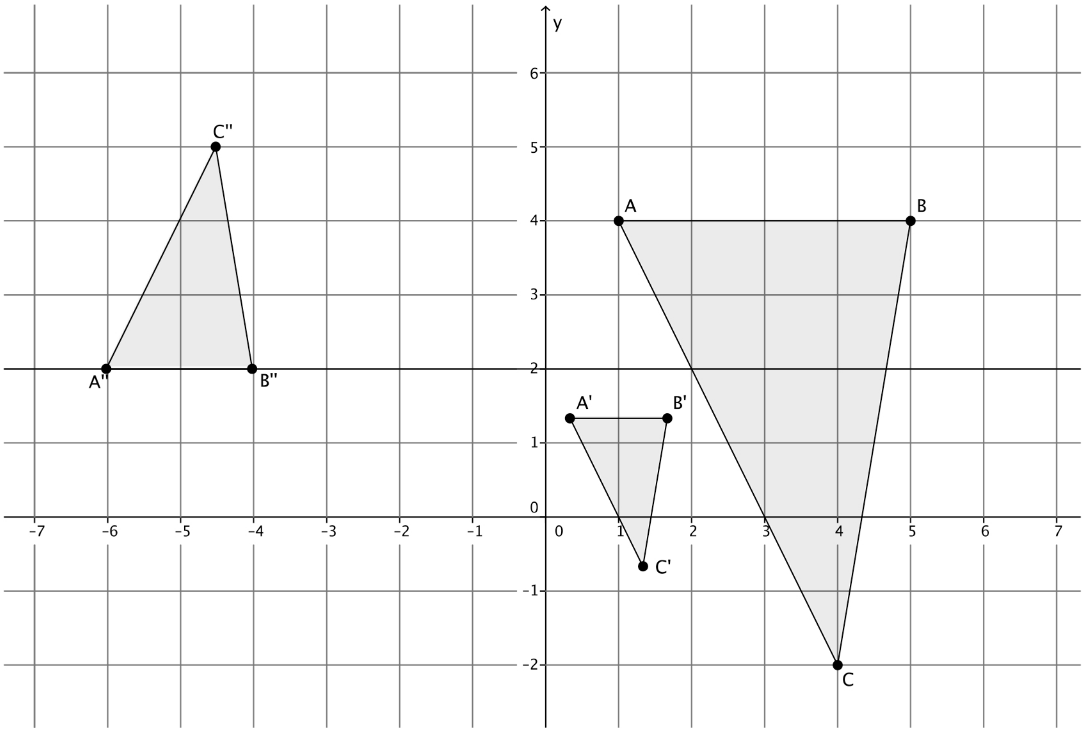


b. Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.

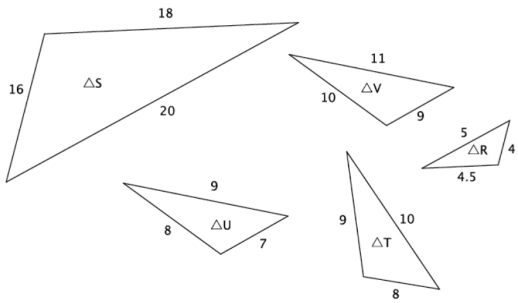


c. In general, is dilation enough to prove that similarity is a transitive relation? Explain.

3. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is $\triangle ABC \sim \triangle A''B''C''$? If so, describe the dilation followed by the congruence that demonstrates the similarity.



Use the diagram below to answer Questions 1 and 2.



1. **Which two triangles, if any, have similarity that is symmetric?**

   $\triangle S \sim \triangle R$ and $\triangle R \sim \triangle S$.

   $\triangle S \sim \triangle T$ and $\triangle T \sim \triangle S$.

   $\triangle T \sim \triangle R$ and $\triangle R \sim \triangle T$.

2. **Which three triangles, if any, have similarity that is transitive?**

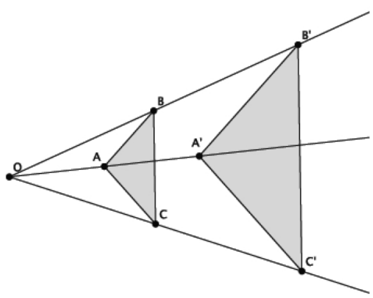
   One possible solution: Since $\triangle S \sim \triangle R$ and $\triangle R \sim \triangle T$, then $\triangle S \sim \triangle T$.

   Note that $\triangle U$ and $\triangle V$ are not similar to each other or any other triangles. Therefore, they should not be in any solution.

Problem Set Sample Solutions

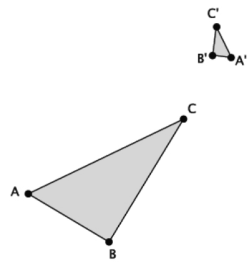
1. **Would a dilation alone be enough to show that similarity is symmetric? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$? Consider the two examples below.**

   a. **Given $\triangle ABC \sim \triangle A'B'C'$. Is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.**



   For these two triangles, a dilation alone is enough to show that if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$. The reason that dilation alone is enough is because both of the triangles have been dilated from the same center. Therefore, to map one onto the other, all that would be required is a dilation.

b. **Given $\triangle ABC \sim \triangle A'B'C'$. Is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.**



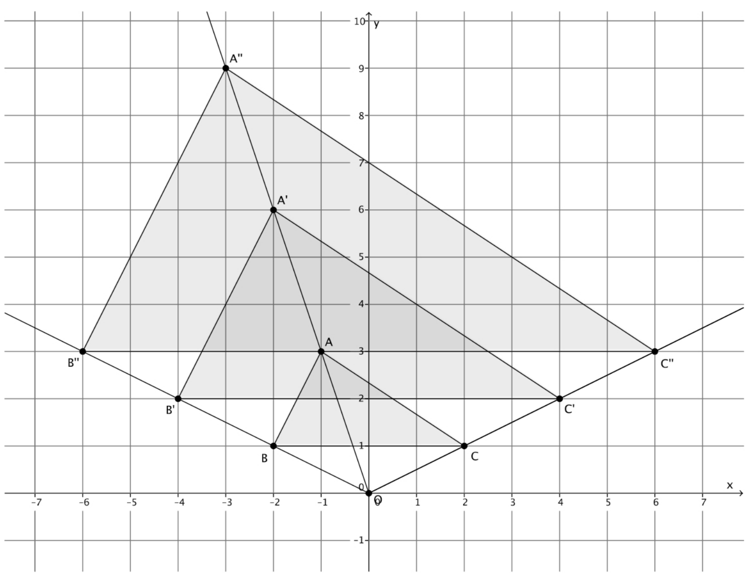
For these two triangles, a dilation alone is not enough to show that if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$. The reason is that a dilation would just make them the same size. It would not show that you could map one of the triangles onto the other. To do that, you would need a sequence of basic rigid motions to demonstrate the congruence.

c. **In general, is dilation enough to prove that similarity is a symmetric relation? Explain.**

No, in general a dilation alone does not prove that similarity is a symmetric relation. In some cases, like part (a), it would be enough, but because we are talking about general cases, we must consider figures that require a sequence of basic rigid motions to map one onto the other. Therefore, in general, to show that there is a symmetric relationship, we must use what we know about similar figures, a dilation followed by a congruence, as opposed to dilation alone.

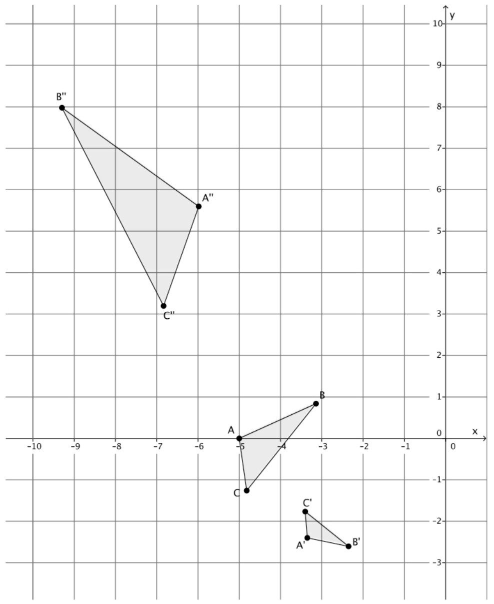
2. **Would a dilation alone be enough to show that similarity is transitive? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$, and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$? Consider the two examples below.**

a. **Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.**



Yes, in this case, we could dilate by different scale factors to show that all three triangles are similar to each other.

b. **Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.**

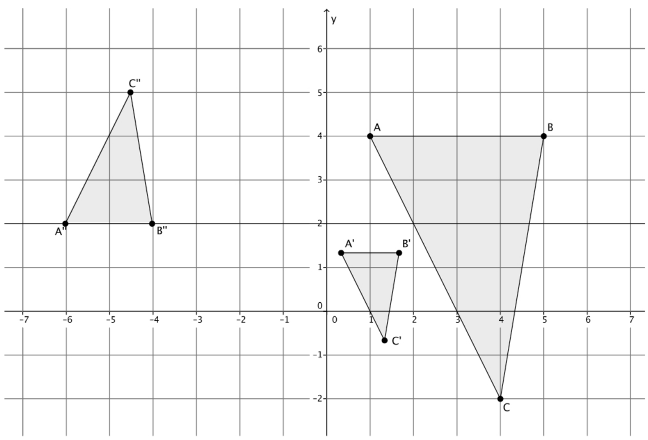


In this case, it would take more than just a dilation to show that all three triangles were similar to one another. Specifically, it would take a dilation followed by a congruence to prove the similarity among the three.

c. **In general, is dilation enough to prove that similarity is a transitive relation? Explain.**

No, in some cases it might be enough, but the general case requires the use of dilation and a congruence. Therefore, to prove that similarity is a transitive relation you must use both a dilation and a congruence.

3. **In the diagram below, $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is $\triangle ABC \sim \triangle A''B''C''$? If so, describe the dilation followed by the congruence that demonstrates the similarity.**



Yes, $\triangle ABC \sim \triangle A''B''C''$ because similarity is transitive. Since $r|AB| = |A''B''|$, then $r \times 4 = 2$, which means $r = \frac{1}{2}$. Then a dilation from the origin by scale factor $r = \frac{1}{2}$ will make $\triangle ABC$ the same size as $\triangle A''B''C''$. Translate the dilated image of $\triangle ABC$ $6\frac{1}{2}$ units to the left, and then reflect across line $A''B''$. The sequence of the dilation and the congruence will map $\triangle ABC$ onto $\triangle A''B''C''$ demonstrating the similarity.