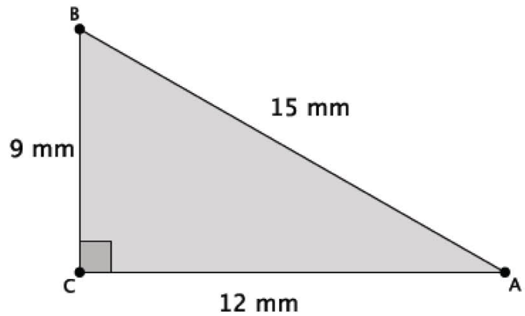
Name ____ Date __________________

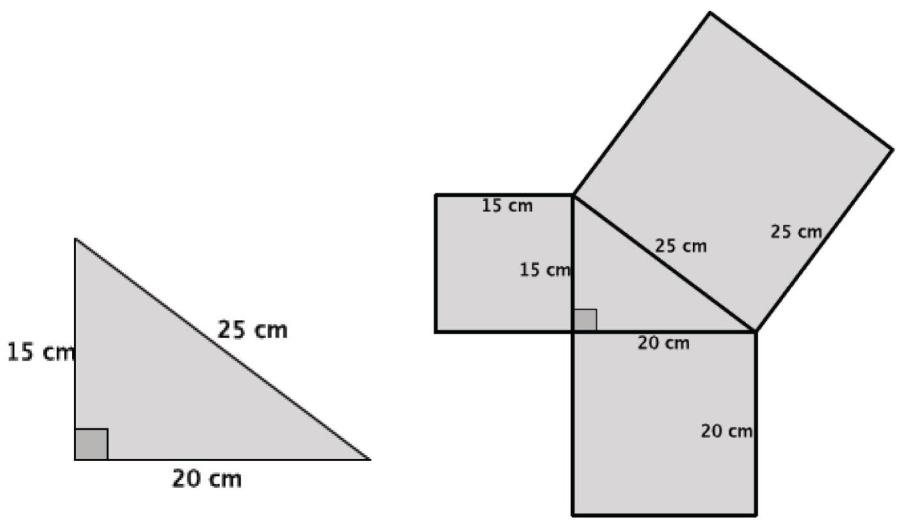
The Pythagorean Theorem, Revisited

Explain a proof of the Pythagorean Theorem in your own words. Use diagrams and concrete examples, as necessary, to support your explanation.

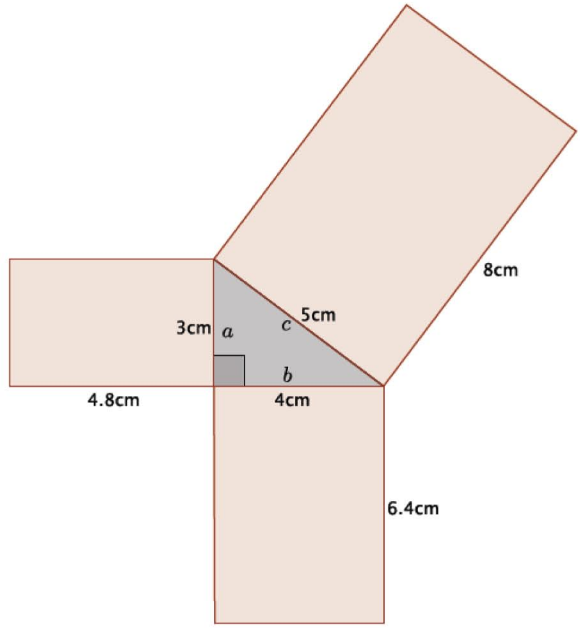
1. For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean Theorem.



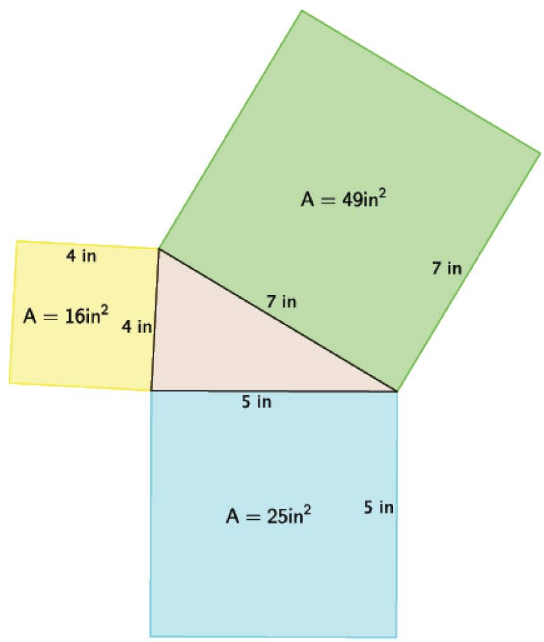
2. For the right triangle shown below, identify and use squares formed by the sides of the triangle to illustrate the Pythagorean Theorem.



3. Reese claimed that any figure can be drawn off the sides of a right triangle and that as long as they are similar figures, then the sum of the areas off of the legs will equal the area off of the hypotenuse. She drew the diagram at right by constructing rectangles off of each side of a known right triangle. Is Reese's claim correct for this example? In order to prove or disprove Reese's claim, you must first show that the rectangles are similar. If they are, then you can use computations to show that the sum of the areas of the figures off of the sides a and b equal the area of the figure off of side c.



4. After learning the proof of the Pythagorean Theorem using areas of squares, Joseph got really excited and tried explaining it to his younger brother using the diagram to the right. He realized during his explanation that he had done something wrong. Help Joseph find his error. Explain what he did wrong.



5. Draw a right triangle with squares constructed off of each side that Joseph can use the next time he wants to show his younger brother the proof of the Pythagorean Theorem.

6. Explain the meaning of the Pythagorean Theorem in your own words.

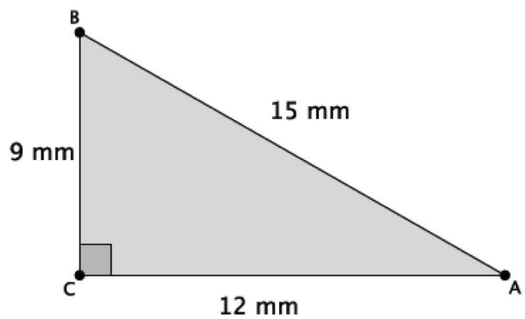
7. Draw a diagram that shown an example illustrating the Pythagorean Theorem.

Explain a proof of the Pythagorean Theorem in your own words. Use diagrams and concrete examples, as necessary, to support your explanation.

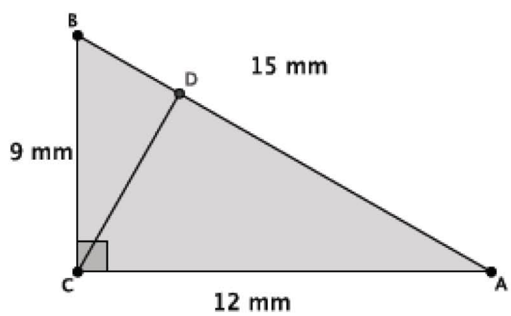
Proofs will vary. The critical parts of the proof that demonstrate proficiency include an explanation of the similar triangles $\triangle ADC$, $\triangle ACB$, and $\triangle CDB$, including a statement about the ratio of their corresponding sides being equal, leading to the conclusion of the proof.

Students apply the concept of similar figures to show the Pythagorean Theorem is true.

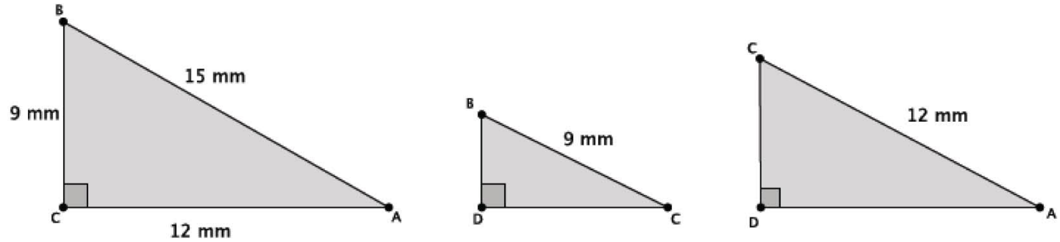
1. **For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean Theorem.**



First, I must draw a segment that is perpendicular to side AB that goes through point C. The point of intersection of that segment and side AB will be marked as point D.



Then, I have three similar triangles: $\triangle ABC$, $\triangle CBD$, $\triangle ACD$, as shown below.



The triangles $\triangle ABC$ and $\triangle CBD$ are similar because each one has a right angle, and they all share $\angle B$. By AA criterion, $\triangle ABC \sim \triangle CBD$. The triangles $\triangle ABC$ and $\triangle ACD$ are similar because each one has a right angle, and they all share $\angle A$. By AA criterion, $\triangle ABC \sim \triangle ACD$. By the transitive property, we also know that $\triangle ACD \sim \triangle CBD$.

Since the triangles are similar, they have corresponding sides that are equal in ratio. For triangles $\triangle ABC$ and $\triangle CBD$:

$$\frac{9}{15} = \frac{|BD|}{9},$$

which is the same as $9^2 = 15(|BD|)$.

For triangles $\triangle ABC$ and $\triangle ACD$:

$$\frac{12}{15} = \frac{|AD|}{12},$$

which is the same as $12^2 = 15(|AD|)$.

Adding these two equations together I get:

$$9^2 + 12^2 = 15(|BD|) + 15(|AD|).$$

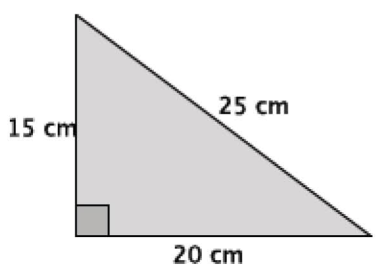
By the distributive property:

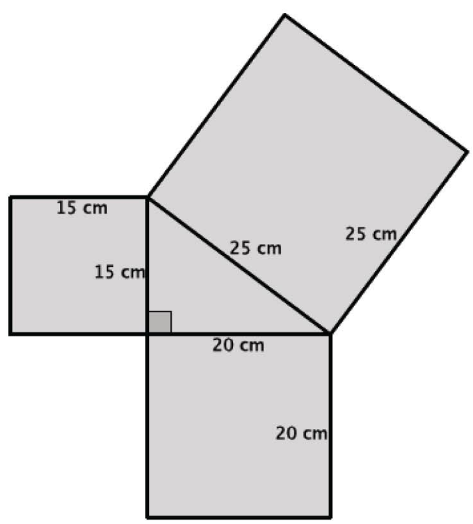
$$9^2 + 12^2 = 15(|BD| + |AD|);$$

however, $|BD| + |AD| = |AC| = 15$; therefore,

$$9^2 + 12^2 = 15(15)$$
$$9^2 + 12^2 = 15^2$$

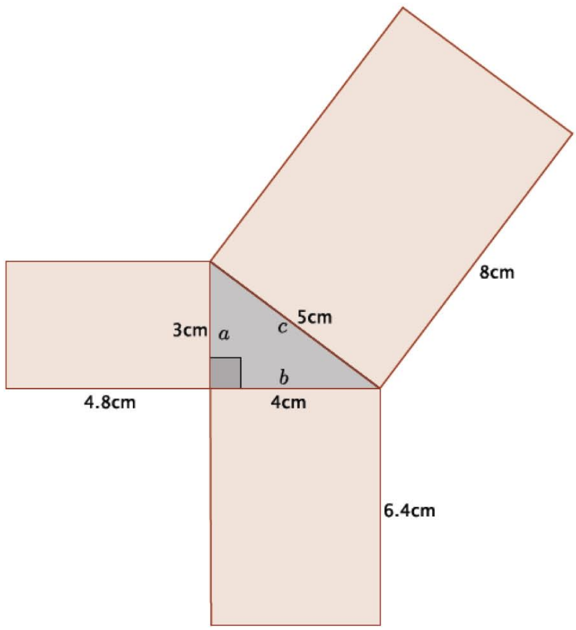
2. **For the right triangle shown below, identify and use squares formed by the sides of the triangle to illustrate the Pythagorean Theorem.**





The sum of the areas of the smallest squares is $15^2 + 20^2 = 625$ cm². The area of the largest square is $25^2 = 625$ cm². The sum of the areas of the squares off of the legs is equal to the area of the square off of the hypotenuse; therefore, $a^2 + b^2 = c^2$.

3. **Reese claimed that any figure can be drawn off the sides of a right triangle and that as long as they are similar figures, then the sum of the areas off of the legs will equal the area off of the hypotenuse. She drew the diagram below by constructing rectangles off of each side of a known right triangle. Is Reese's claim correct for this example? In order to prove or disprove Reese's claim, you must first show that the rectangles are similar. If they are, then you can use computations to show that the sum of the areas of the figures off of the sides $a$and b equal the area of the figure off of side c.**



The rectangles are similar because their corresponding side lengths are equal in scale factor. That is, if we compare the longest side of the rectangle to the side with the same length as the right triangle sides, we get the ratios

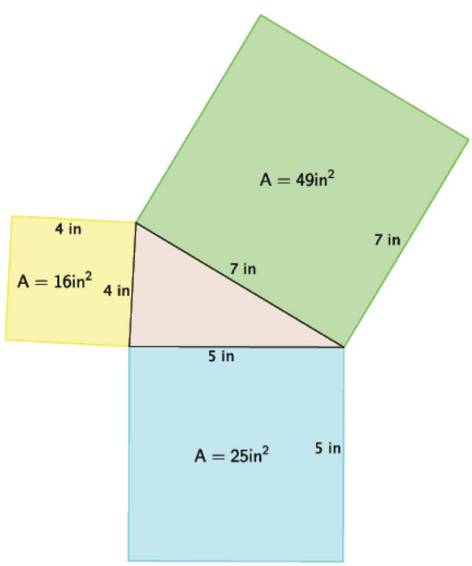
$$\frac{4.8}{3} = \frac{6.4}{4}$$
$$= \frac{8}{5} = 1.6$$

Since the corresponding sides were all equal to the same constant, then we know we have similar rectangles. The areas of the smaller rectangles are $14.4\ cm^2$ and $25.6\ cm^2$, and the area of the largest rectangle is $40\ cm^2$. The sum of the smaller areas is equal to the larger area:

$$14.4 + 24.6 = 40$$
$$40 = 40$$

Therefore, we have shown that the sum of the areas of the two smaller rectangles is equal to the area of the larger rectangle, and Reese's claim is correct.

4. **After learning the proof of the Pythagorean Theorem using areas of squares, Joseph got really excited and tried explaining it to his younger brother. He realized during his explanation that he had done something wrong. Help Joseph find his error. Explain what he did wrong.**



Based on the proof shown in class, we would expect the sum of the two smaller areas to be equal to the sum of the larger area, i.e., $16 + 25$ should equal 49. However, $16 + 25 = 41$. Joseph correctly calculated the areas of each square, so that was not his mistake. His mistake was claiming that a triangle with sides lengths of 4, 5, and 7 was a right triangle. We know that the Pythagorean Theorem only works for right triangles. Considering the converse of the Pythagorean Theorem, when we use the given side lengths, we do not get a true statement.

$$4^2 + 5^2 = 7^2$$
$$16 + 25 = 49$$
$$41 \neq 49$$

Therefore, the triangle Joseph began with is not a right triangle, so it makes sense that the areas of the squares were not adding up like we expected.

5. **Draw a right triangle with squares constructed off of each side that Joseph can use the next time he wants to show his younger brother the proof of the Pythagorean Theorem.**

Answers will vary. Verify that students begin, in fact, with a right triangle and do not make the same mistake that Joseph did. Consider having students share their drawings and explanations of the proof in future class meetings.

6. **Explain the meaning of the Pythagorean Theorem in your own words.**

If a triangle is a right triangle, then the sum of the squares of the legs will be equal to the square of the hypotenuse. Specifically, if the leg lengths are a and b, and the hypotenuse is length c, then for right triangles $a^2 + b^2 = c^2$.

7. **Draw a diagram that shows an example illustrating the Pythagorean Theorem.**

Diagrams will vary. Verify that students draw a right triangle with side lengths that satisfy the Pythagorean Theorem.

Diagrams referenced in scaffolding boxes can be reproduced for use student use.