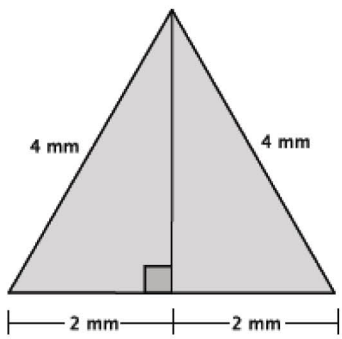
Name _____ Date _____

Applications of the Pythagorean Theorem

1. Use the diagram of the equilateral triangle shown below to answer the following questions. Show work that leads to your answers.



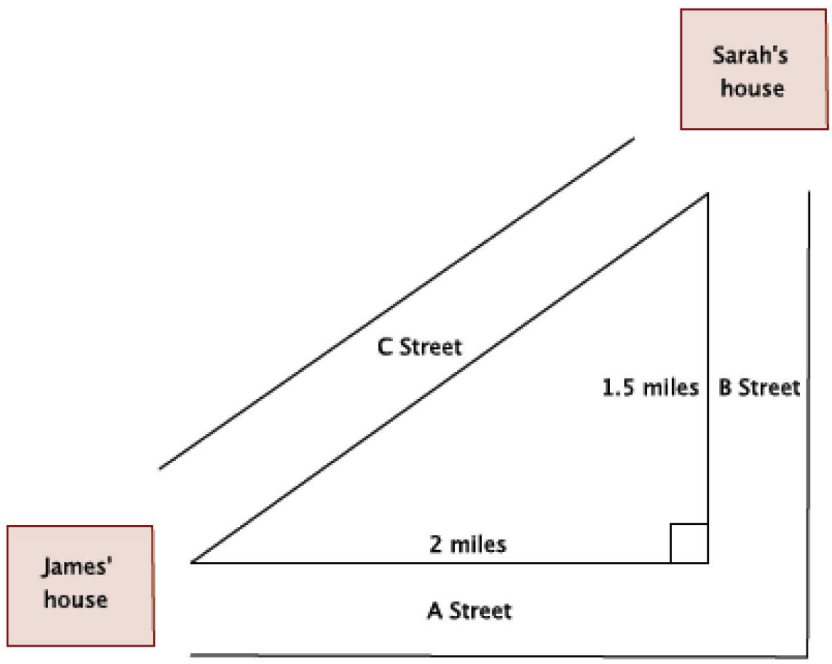
   a. What is the perimeter of the triangle?

   b. What is the height, h, of the equilateral triangle? Write an exact answer using a square root and approximate answer rounded to the tenths place.

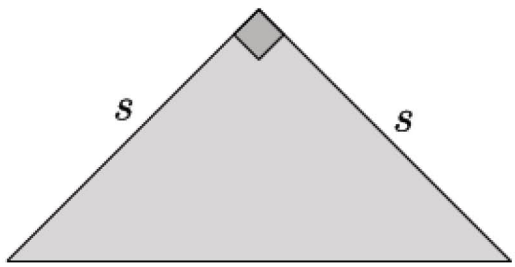
   c. Using the approximate height found in part (b), estimate the area of the equilateral triangle.

1. A 70” TV is advertised on sale at a local store. What are the length and width of the television?

2. There are two paths that one can use to go from Sarah’s house to James’ house. One way is to take C Street, and the other way requires you to use A Street and B Street. How much shorter is the direct path along C Street?



3. An isosceles right triangle refers to a right triangle with equal leg lengths, s, as shown below.

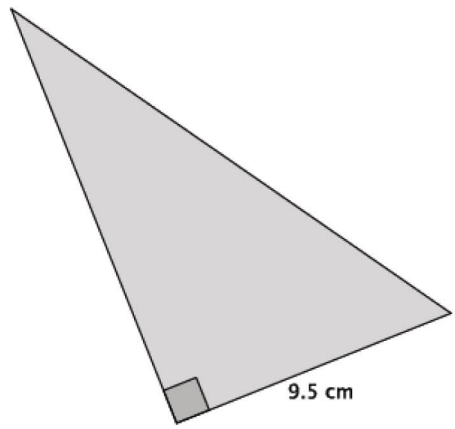


What is the length of the hypotenuse of an isosceles right triangle with a leg length of 9 cm? Write an exact answer using a square root and an approximate answer rounded to the tenths place.

4. The area of the right triangle shown at right is 66.5 cm^2.

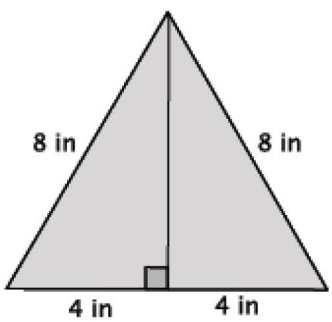
   a. What is the height of the triangle?

   b. What is the perimeter of the right triangle? Round your answer to the tenths place.

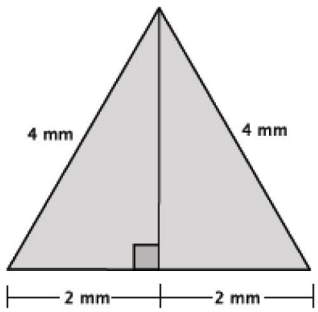


5. What is the distance between points $(1, 9)$ and $(-4, -1)$? Round your answer to the tenths place.

6. An equilateral triangle is shown below. Determine the area of the triangle. Round your answer to the tenths place.



1. Use the diagram of the equilateral triangle shown below to answer the following questions. Show work that leads to your answers.



a. What is the perimeter of the triangle?

$4 + 4 + 4 = 12$

The perimeter is 12 mm.

b. What is the height, h, of the equilateral triangle? Write an exact answer using a square root and approximate answer rounded to the tenths place.

Using the fact that the height is one leg length of a right triangle, and I know the hypotenuse is 4 mm and the other leg length is 2 mm, I can use the Pythagorean Theorem to find h.

$$\begin{aligned} 2^2 + h^2 &= 4^2 \\ 4 + h^2 &= 16 \\ 4 - 4 + h^2 &= 16 - 4 \\ h^2 &= 12 \\ h &= \sqrt{12} \\ h &= \sqrt{4 \times 3} \\ h &= \sqrt{4} \times \sqrt{3} \\ h &= 2\sqrt{3} \end{aligned}$$

The number $\sqrt{3}$ is between 1 and 2. In the sequence of tenths, it is between 1.7 and 1.8 because $1.7^2 < \left(\sqrt{3}\right)^2 < 1.8^2$. In the sequence of hundredths, it is between 1.73 and 1.74, which means it would round to 1.7. Then $2 \times 1.7 = 3.4$ mm is the approximate length of the hypotenuse and $\sqrt{12} = 2\sqrt{3}$ cm is the exact length.

c. Using the approximate height found in part (b), estimate the area of the equilateral triangle.

$$\begin{aligned} A &= \frac{bh}{2} \\ A &= \frac{4(3.4)}{2} \\ A &= \frac{13.6}{2} \\ A &= 6.8 \end{aligned}$$

The approximate area of the equilateral triangle is 6.8 mm^2.

Students continue applying the Pythagorean Theorem to solve real-world and mathematical problems.

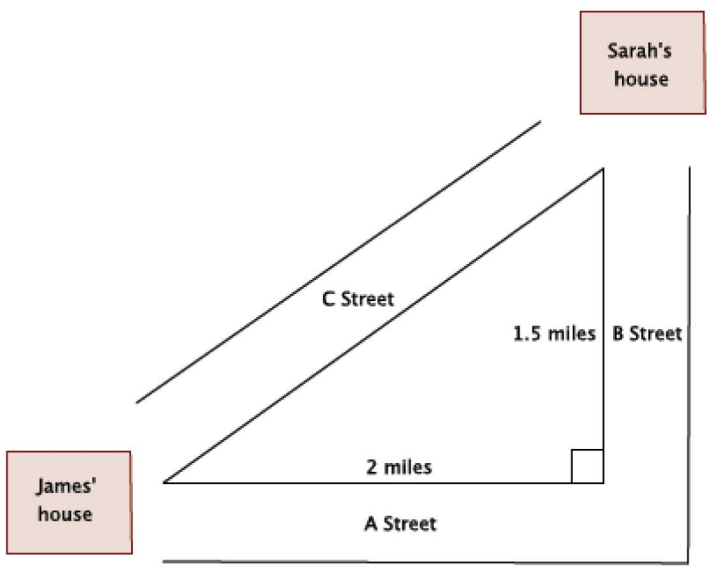
1. **A 70" TV is advertised on sale at a local store. What are the length and width of the television?**

   The TV is in the ratio of $4:3$ and has measurements of $4x:3x$, where x is the scale factor of enlargement.

$$\begin{aligned}(3x)^2+(4x)^2&=70^2\\ 9x^2+16x^2&=4,900\\ 25x^2&=4,900\\ \frac{25x^2}{25}&=\frac{4,900}{25}\\ x^2&=196\\ \sqrt{x^2}&=\sqrt{196}\\ x&=14\end{aligned}$$

   The length of the TV is $4\times 14=56$ inches and the width is $3\times 14=42$ inches.

2. **There are two paths that one can use to go from Sarah's house to James' house. One way is to take C Street, and the other way requires you to use A Street and B Street. How much shorter is the direct path along C Street?**

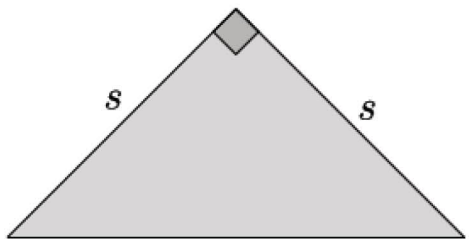


   Let c represent the hypotenuse of the right triangle.

$$\begin{aligned}2^2+1.5^2&=c^2\\ 4+2.25&=c^2\\ 6.25&=c^2\\ \sqrt{6.25}&=\sqrt{c^2}\\ 2.5&=c\end{aligned}$$

   The path using A Street and B Street is 3.5 miles. The path along C Street is 2.5 miles. The path along C Street is exactly 1 mile shorter than the path along A Street and B Street.

3. An isosceles right triangle refers to a right triangle with equal leg lengths, s, as shown below.



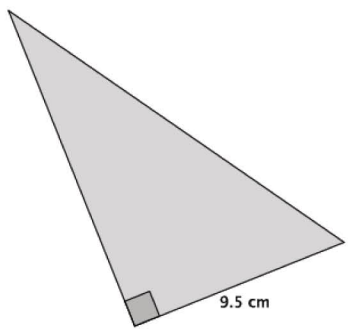
What is the length of the hypotenuse of an isosceles right triangle with a leg length of 9 cm? Write an exact answer using a square root and an approximate answer rounded to the tenths place.

Let c be the hypotenuse of the isosceles triangle.

$$\begin{aligned} 9^2 + 9^2 &= c^2 \\ 81 + 81 &= c^2 \\ 162 &= c^2 \\ \sqrt{162} &= \sqrt{c^2} \\ \sqrt{81 \times 2} &= c \\ \sqrt{81} \times \sqrt{2} &= c \\ 9\sqrt{2} &= c \end{aligned}$$

The number $\sqrt{2}$ is between 1 and 2. In the sequence of tenths, it is between 1.4 and 1.5 because $1.4^2 < \left(\sqrt{2}\right)^2 < 1.5^2$. Since the number 2 is closer to 1.4^2 than 1.5^2, it would round to 1.4. Then $9 \times 1.4 = 12.6$ cm is the approximate length of the hypotenuse, and $9\sqrt{2}$ cm is the exact length.

4. The area of the right triangle shown below is 66.5 cm².



a. What is the height of the triangle?

$$\begin{aligned} A &= \frac{bh}{2} \\ 66.5 &= \frac{9.5h}{2} \\ 133 &= 9.5h \\ \frac{133}{9.5} &= \frac{9.5h}{9.5} \\ 14 &= h \end{aligned}$$

b. What is the perimeter of the right triangle? Round your answer to the tenths place.

Let c represent the length of the hypotenuse.

$$\begin{aligned} 9.5^2 + 14^2 &= c^2 \\ 90.25 + 196 &= c^2 \\ 286.25 &= c^2 \\ \sqrt{286.25} &= \sqrt{c^2} \\ \sqrt{286.25} &= c \end{aligned}$$

The number $\sqrt{286.25}$ is between 16 and 17. In the sequence of tenths, the number is between 16.9 and 17 because $16.9^2 < \left(\sqrt{286.25}\right)^2 < 17^2$. Since 286.25 is closer to 16.9^2 than 17^2, then the approximate length of the hypotenuse is 16.9 cm.

The perimeter of the triangle is $9.5 + 14 + 16.9 = 40.4$ cm.

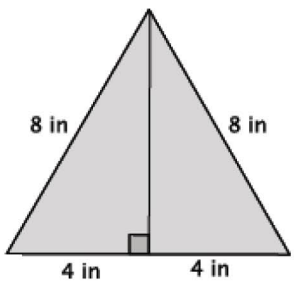
5. **What is the distance between points $(1, 9)$ and $(-4, -1)$? Round your answer to the tenths place.**

Let c represent the distance between the points.

$$\begin{aligned} 10^2 + 5^2 &= c^2 \\ 100 + 25 &= c^2 \\ 125 &= c^2 \\ \sqrt{125} &= \sqrt{c^2} \\ \sqrt{125} &= c \\ 11.2 &\approx c \end{aligned}$$

The distance between the points is approximately 11.2 units.

6. **An equilateral triangle is shown below. Determine the area of the triangle. Round your answer to the tenths place.**



Let h represent the height of the triangle.

$$\begin{aligned} 4^2 + h^2 &= 8^2 \\ 16 + h^2 &= 64 \\ h^2 &= 48 \\ \sqrt{h^2} &= \sqrt{48} \\ h &= \sqrt{48} \\ h &\approx 6.9 \end{aligned}$$

$$A = \frac{8(6.9)}{2} = 4(6.9) = 27.6$$

The area of the triangle is 27.6 in^2.