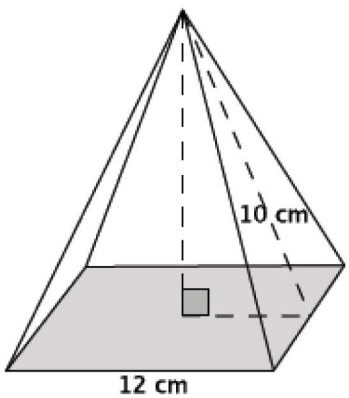
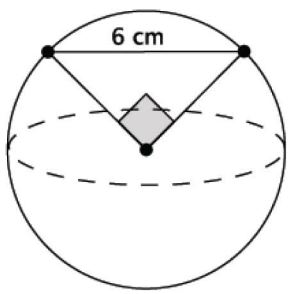
Name _____ Date _____

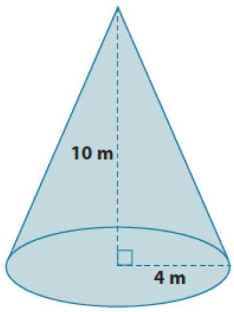
Cones and Spheres

Which has the larger volume? Give an approximate answer rounded to the tenths place.

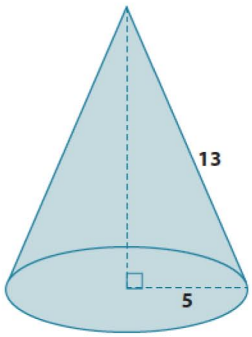




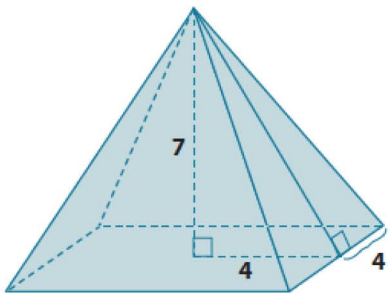
1. What is the lateral length of the cone shown below? Give an approximate answer rounded to the tenths place.



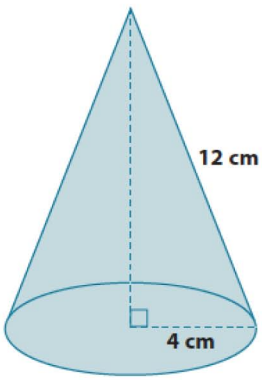
2. What is the volume of the cone shown below? Give an exact answer.



3. Determine the volume and surface area of the pyramid shown below. Give exact answers.

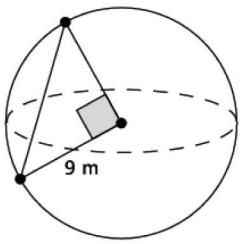


4. Alejandra computed the volume of the cone shown below as 64π cm^2. Her work is shown below. Is she correct? If not, explain what she did wrong and calculate the correct volume of the cone. Give an exact answer.

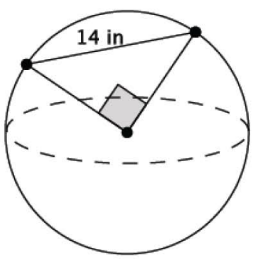


$$V = \frac{1}{3}\pi(4^2)(12)$$
$$= \frac{16(12)\pi}{3}$$
$$= 64\pi$$
$$= 64\pi\ cm^3$$

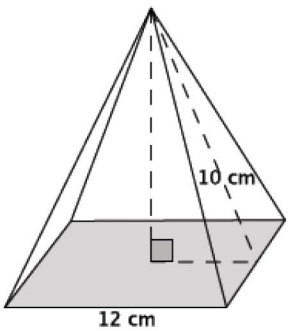
5. What is the length of the chord of the sphere shown below? Give an exact answer using a square root.

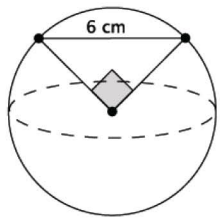


6. What is the volume of the sphere shown below? Give an exact answer using a square root.



Which has the larger volume? Give an approximate answer rounded to the tenths place.





Let h represent the height of the square cone.

$$h^2 + 6^2 = 10^2$$
$$h^2 + 36 = 100$$
$$h^2 = 64$$
$$h = 8$$

$$V = \frac{1}{3}(12^2)8$$
$$V = \frac{1}{3}(144)8$$
$$V = 384$$

The volume of the square pyramid is 384 cm³.

Let r represent the radius of the sphere.

$$r^2 + r^2 = 6^2$$
$$2r^2 = 36$$
$$r^2 = 18$$
$$\sqrt{r^2} = \sqrt{18}$$
$$r = \sqrt{3^2 \times 2}$$
$$r = 3\sqrt{2}$$

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(3\sqrt{2}\right)^3$$
$$= \frac{4}{3}\pi 3^3\left(\sqrt{2}\right)^3$$
$$= \frac{4}{3}\pi 27\left(\sqrt{8}\right)$$
$$= \frac{4}{3}\pi 27\left(\sqrt{2^2 \times 2}\right)$$
$$= \frac{4}{3}\pi 27(2)\left(\sqrt{2}\right)$$
$$= 72\pi\sqrt{2}$$

The volume of the sphere is $72\pi\sqrt{2}$ cm³.

The number $\sqrt{2}$ is between 1 and 2. In the sequence of tenths, it is between 1.4 and 1.5. Since 2 is closer to 1.4^2 than 1.5^2, then the number is approximately 1.4.

We know from previous lessons, we can estimate $\pi = 3.1$.

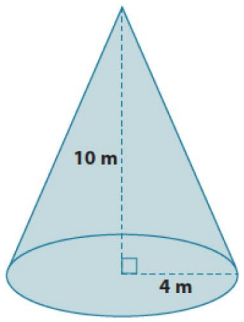
Then the approximate volume of the sphere is

$$V = 72(1.4)3.1$$
$$V = 312.48$$

Therefore, the volume of the square cone is greater.

Students use the Pythagorean Theorem to solve mathematical problems in three dimensions.

1. What is the lateral length of the cone shown below? Give an approximate answer rounded to the tenths place.



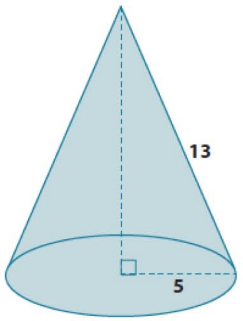
Let c be the lateral length

$$10^2 + 4^2 = c^2$$
$$100 + 16 = c^2$$
$$116 = c^2$$
$$\sqrt{116} = \sqrt{c^2}$$
$$\sqrt{116} = c$$

The number $\sqrt{116}$ is between 10 and 11. In the sequence of tenths, it is between 10.7 and 10.8. Since 116 is closer to 10.8^2 than 10.7^2 then the approximate value of the number is 10.8.

The lateral length of the cone is approximately 10.8 m.

2. What is the volume of the cone shown below? Give an exact answer.



Let h represent the height of a cone.

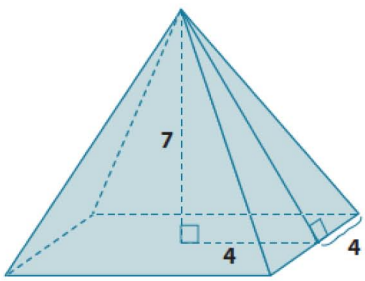
$$5^2 + h^2 = 13^2$$
$$25 + h^2 = 169$$
$$h^2 = 144$$
$$\sqrt{h^2} = \sqrt{144}$$
$$h = 12$$

The height of the cone is 12 units.

$$V = \frac{1}{3}\pi\, 25(12)$$
$$= 100\pi$$

The volume of the cone is 100π units^3.

3. Determine the volume and surface area of the pyramid shown below. Give exact answers.



$$V = \frac{1}{3}(64)(7)$$
$$= \frac{448}{3}$$

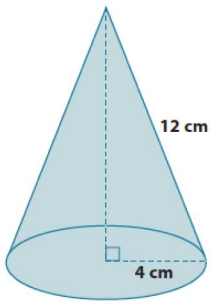
The volume of the pyramid is $\frac{448}{3}$ units^3.

Let c represent the lateral length.

$$7^2 + 4^2 = c^2$$
$$49 + 16 = c^1$$
$$65 = c^2$$
$$\sqrt{65} = \sqrt{c^2}$$
$$\sqrt{65} = c$$

The base area is 64 units^2, and the four faces are each $\frac{8\sqrt{65}}{2} = 4\sqrt{65}$, so the total area of the faces is $4 \times 4\sqrt{65} = 16\sqrt{65}$. The surface area is $64 + 16\sqrt{65}$ units^2.

4. Alejandra computed the volume of the cone shown below as 64π cm^2. Her work is shown below. Is she correct? If not, explain what she did wrong and calculate the correct volume of the cone. Give an exact answer.



$$V = \frac{1}{3}\pi(4^2)(12)$$
$$= \frac{16(12)\pi}{3}$$
$$= 64\pi$$
$$= 64\pi\ cm^3$$

Alejandra's work is incorrect. She used the lateral length instead of the height of the cone to compute volume.

Let h represent the height.

$$4^2 + h^2 = 12^2$$
$$16 + h^2 = 144$$
$$h^2 = 128$$
$$\sqrt{h^2} = \sqrt{128}$$
$$h = \sqrt{128}$$
$$h = \sqrt{8^2 \times 2}$$
$$h = 8\sqrt{2}$$

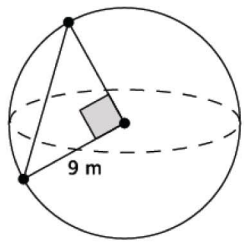
$$V = \frac{1}{3}\pi 4^2(8\sqrt{2})$$
$$V = \frac{1}{3}\pi 128\sqrt{2}$$
$$V = \frac{128\sqrt{2}}{3}\pi$$

The volume of the cone is $\frac{128\sqrt{2}}{3}\pi$ cm^3.

5. What is the length of the chord of the sphere shown below? Give an exact answer using a square root.

Let c represent the length of the chord.

$$9^2 + 9^2 = c^2$$
$$81 + 81 = c^2$$
$$162 = c^2$$
$$\sqrt{162} = \sqrt{c^2}$$
$$\sqrt{162} = c$$
$$\sqrt{9^2 \times 2} = c$$
$$9\sqrt{2} = c$$



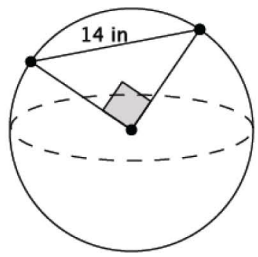
The length of the chord is $\sqrt{162} = 9\sqrt{2}$ m.

6. What is the volume of the sphere shown below? Give an exact answer using a square root.

Let r represent the radius.

$$r^2 + r^2 = 14^2$$
$$2r^2 = 196$$
$$r^2 = 98$$
$$\sqrt{r^2} = \sqrt{98}$$
$$r = \sqrt{7^2 \times 2}$$
$$r = 7\sqrt{2}$$

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(7\sqrt{2}\right)^3$$
$$= \frac{4}{3}\pi 343\left(\sqrt{8}\right)$$
$$= \frac{4}{3}\pi 343\left(2\sqrt{2}\right)$$
$$= \frac{2,744\sqrt{2}}{3}\pi$$



The volume of the sphere is $\frac{4}{3}\pi\left(\sqrt{98}\right)^3 = \frac{2,744\sqrt{2}}{3}\pi$ in^3.