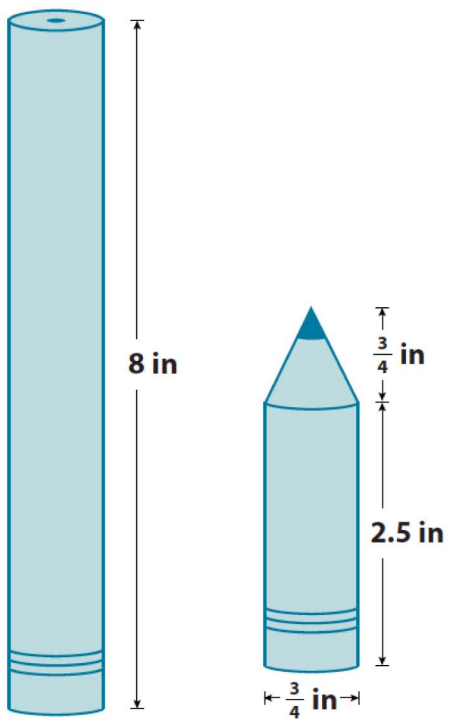Name _____   Date _____

Volume of Composite Solids

Andrew bought a new pencil like the one shown below on the left. He used the pencil every day in his math class for a week, and now his pencil looks like the one shown below on the right. How much of the pencil, in terms of volume, did he use?
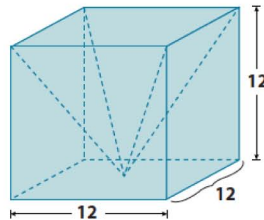
Note: Figures not drawn to scale.

1. What volume of sand would be required to completely fill up the hourglass shown below? Note: 12m is the height of the truncated cone, not the lateral length of the cone.
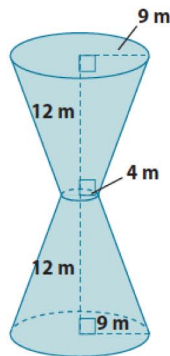
2. a. Write an expression that can be used to find the volume of the prism with the pyramid portion removed. Explain what each part of your expression represents.
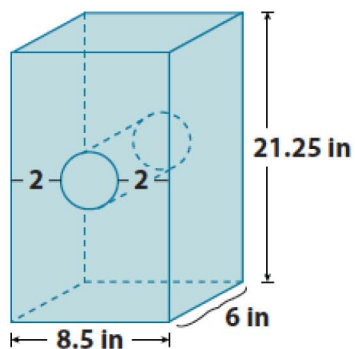

    b. What is the volume of the prism shown above with the pyramid portion removed?

3. a. Write an expression that can be used to find the volume of the funnel shown below. Explain what each part of your expression represents.
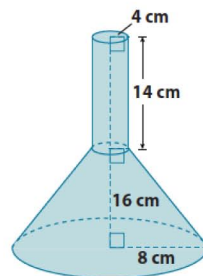
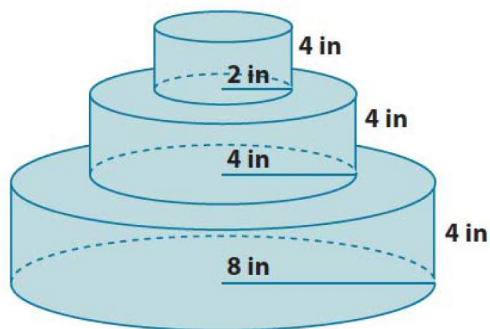   b. Determine the exact volume of the funnel shown above.

4. What is the approximate volume of the rectangular prism with a cylindrical hole shown below? Use 3.14 for π. Round your answer to the tenths place.

21.25 in

2 — 2

6 in

← 8.5 in →

5. A layered cake is being made to celebrate the end of the school year. What is the exact total volume of the cake shown below?

4 in

2 in

4 in

4 in

8 in

4 in

Andrew bought a new pencil like the one shown below on the left. He used the pencil every day in his math class for a week, and now his pencil looks like the one shown below on the right. How much of the pencil, in terms of volume, did he use?
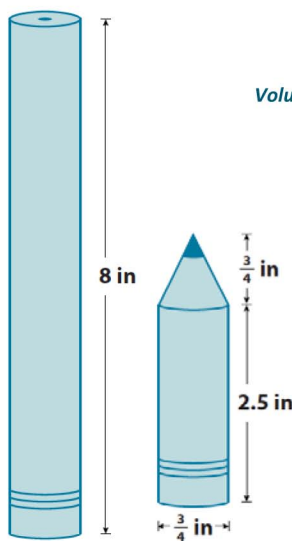
$$V = \pi(0.375)^2 8$$
$$V = 1.125\pi$$

Volume of the pencil at the beginning of the week was 1.125π in³.

$$V = \pi(0.375)^2 2.5$$
$$V = 0.3515\pi$$

The volume of the cylindrical part of the pencil is 0.3515π in³.

$$V = \frac{1}{3}\pi(0.375)^2(0.75)$$
$$V = \frac{0.1054}{3}\pi$$
$$V = 0.0351\pi$$

The volume of the cone part of the pencil is 0.0351π in³.

$$0.3515\pi + 0.0351\pi = (0.3515 + 0.0351)\pi = 0.3866\pi.$$

The total volume of the pencil after a week is 0.3866π in³.

$$1.125\pi - 0.3866\pi = (1.125 - 0.3866)\pi = 0.7384\pi.$$

In one week, Andrew used 0.7384π in³ of the pencil's total volume.

8 in

$\frac{3}{4}$ **in**

2.5 in

$\frac{3}{4}$ **in**

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1. What volume of sand would be required to completely fill up the hourglass shown below? Note: 12m is the height of the truncated cone, not the lateral length of the cone.

Let x represent the height of the portion of the cone that has been removed.

$$\frac{4}{9} = \frac{x}{x + 12}$$
$$4(x + 12) = 9x$$
$$4x + 48 = 9x$$
$$48 = 5x$$
$$\frac{48}{5} = x$$
$$9.6 = x$$

9 m

12 m

4 m

12 m

9 m

The volume of the removed cone is   *The volume of the cone is*

$$V = \frac{1}{3}\pi 4^2(9.6)$$
$$= \frac{153.6}{3}\pi$$

$$V = \frac{1}{3}\pi 9^2(21.6)$$
$$= \frac{1,749.6}{3}\pi$$

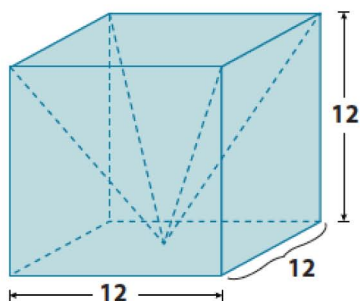*The volume of one truncated cone is*

$$\frac{1,749.6}{3}\pi - \frac{153.6}{3}\pi = \left(\frac{1,749.6}{3} - \frac{153.6}{3}\right)\pi$$
$$= \frac{1,596}{3}\pi$$
$$= 532\,\pi$$

The volume of sand needed to fill the hourglass is $1,064\pi$ m³.
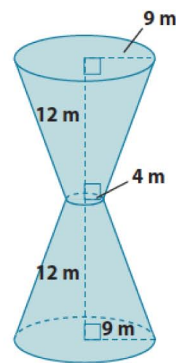
2. **a.** Write an expression that can be used to find the volume of the prism with the pyramid portion removed. Explain what each part of your expression represents.

The expression 12^3 is the volume of the cube and $\frac{1}{3}(12^3)$ is the volume of the pyramid. Since the pyramid's volume is being removed from the cube, then we subtract the volume of the pyramid from the cube.

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b. What is the volume of the prism shown above with the pyramid portion removed?
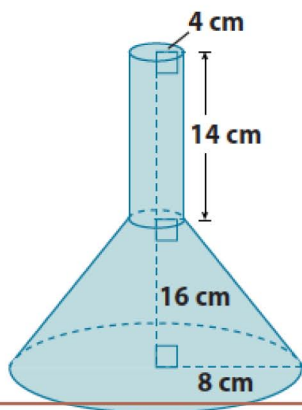
The volume of the prism is

$$V = 12^3$$
$$= 1,728$$

The volume of the pyramid is

$$V = \frac{1}{3}(1,728)$$
$$= 576$$

The volume of the prism with the pyramid removed is $1,152$ units³.

3. **a.** Write an expression that can be used to find the volume of the funnel shown below. Explain what each part of your expression represents.

The expression $\pi 4^2(14)$ represents the volume of the cylinder. The expression $\left(\frac{1}{3}\pi 8^2(x+16) - \frac{1}{3}\pi 4^2(x)\right)$ represents the volume of the truncated cone. The x represents the unknown height of the smaller cone. When the volume of the cylinder is added to the volume of the truncated cone, then we will have the volume of the funnel shown.

The expression $\pi 4^2(14)$ represents the volume of the cylinder. The expression $\left(\frac{1}{3}\pi 8^2(x+16) - \frac{1}{3}\pi 4^2(x)\right)$ represents the volume of the truncated cone. The x represents the unknown height of the smaller cone. When the volume of the cylinder is added to the volume of the truncated cone, then we will have the volume of the funnel shown.

b. Determine the exact volume of the funnel shown above.

The volume of the cylinder is

$$V = \pi 4^2 (14)$$
$$= 224\pi$$

Let x be the height of the cone that has been removed.

$$\frac{4}{8} = \frac{x}{x+16}$$
$$4(x + 16) = 8x$$
$$4x + 64 = 8x$$
$$64 = 4x$$
$$16 = x$$

The volume of the small cone is

$$V = \frac{1}{3}\pi 4^2 (16)$$
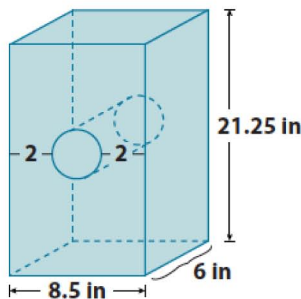$$= \frac{256}{3}\pi$$

The volume of the large cone is

$$V = \frac{1}{3}\pi 8^2 (32)$$
$$= \frac{2,048}{3}\pi$$

The volume of the truncated cone is

$$\frac{2,048}{3}\pi - \frac{256}{3}\pi = \left(\frac{2,048}{3} - \frac{256}{3}\right)\pi$$
$$= \frac{1,792}{3}\pi$$

The volume of the funnel is $224\pi + \frac{1,792}{3}\pi = 821\frac{1}{3}\pi$ cm³.

4. What is the approximate volume of the rectangular prism with a cylindrical hole shown below? Use 3.14 for π. Round your answer to the tenths place.
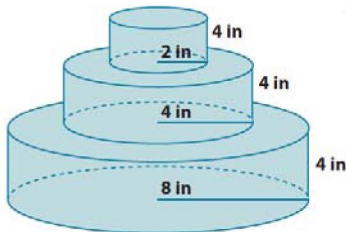

The volume of the prism is

$$V = 8.5(6)(21.25)$$
$$= 1,083.75$$

The volume of the cylinder is

$$V = \pi(2.25)^2(6)$$
$$= 30.375\pi$$
$$\approx 95.3775$$

The volume of the prism with the cylindrical hole is
$$1,083.75 - 95.3775 = 988.3725 \approx 988.4 \text{ in}^3.$$

5. A layered cake is being made to celebrate the end of the school year. What is the exact total volume of the cake shown below?

The bottom layer's volume is

$$V = 8^2\pi(4)$$
$$= 256\pi$$

The middle layer's volume is

$$V = 4^2\pi(4)$$
$$= 64\pi$$

The top layer's volume is

$$V = 2^2\pi(4)$$
$$= 16\pi$$

The total volume of the cake is
$$256\pi + 64\pi + 16\pi = (256 + 64 + 16)\pi = 336\pi \text{ in}^3$$