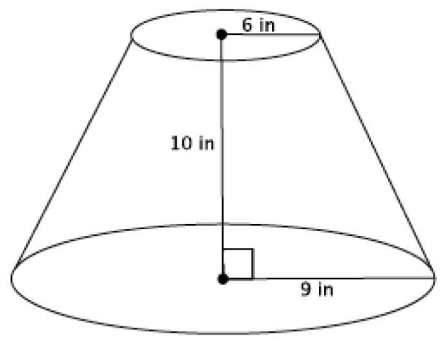
Name ____ Date ____________________

Truncated Cones

Find the volume of the truncated cone.



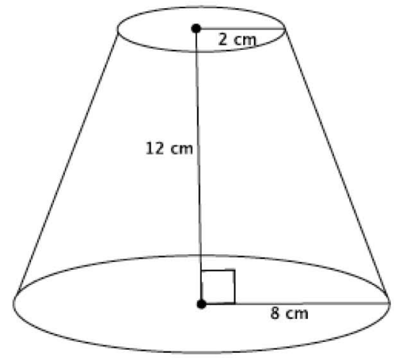
a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.

b. Solve your proportion to determine the height of the cone that has been removed.

c. Write an expression that can be used to determine the volume of the truncated cone. Explain what each part of the expression represents.

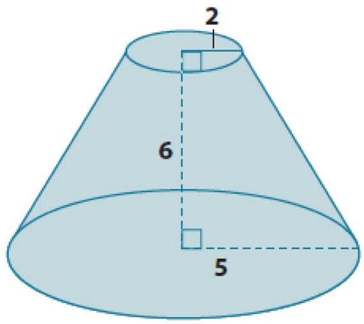
d. Calculate the volume of the truncated cone.

1. Find the volume of the truncated cone.

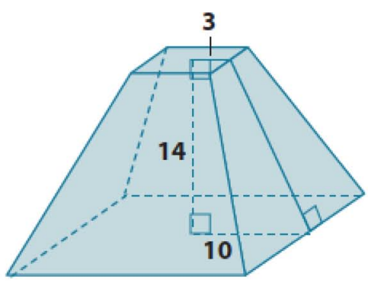


   a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.
   b. Solve your proportion to determine the height of the cone that has been removed.
   c. Show a fact about the volume of the truncated cone using an expression. Explain what each part of the expression represents.
   d. Calculate the volume of the truncated cone.

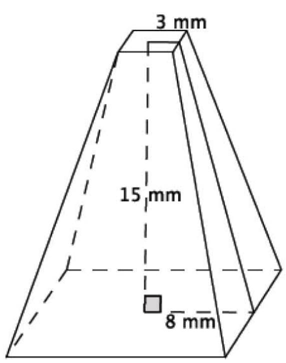
2. Find the volume of the truncated cone.



3. Find the volume of the truncated pyramid with a square base.

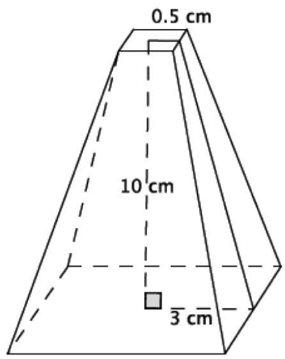


4. Find the volume of the truncated pyramid with a square base. Note: 3 mm is the distance from the center to the edge of the square at the top of the figure.

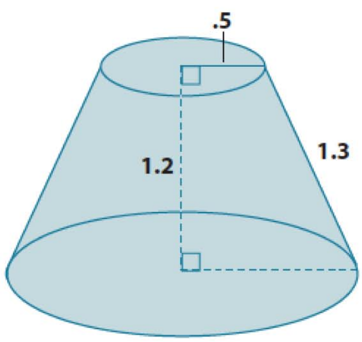


5. Find the volume of the truncated pyramid with a square base. Note: 0.5 cm is the distance from the center to the edge of the square at the top of the figure.

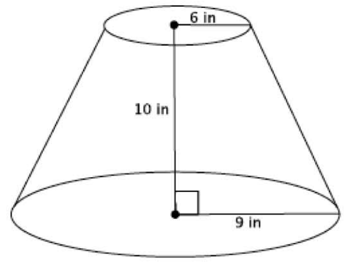
6. Explain how to find the volume of a truncated cone.



7. Challenge: Find the volume of the truncated cone.



Find the volume of the truncated cone.



a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.

$$\frac{6}{9}=\frac{x}{x+10}$$

Let x represent the height of the small cone. Then $x+10$ is the height of the large cone. Then 6 is the base radius of the small cone, and the 9 is the base radius of the large cone.

b. Solve your proportion to determine the height of the cone that has been removed.

$$\begin{aligned}6(x+10)&=9x\\6x+60&=9x\\60&=3x\\20&=x\end{aligned}$$

c. Write an expression that can be used to determine the volume of the truncated cone. Explain what each part of the expression represents.

$$\frac{1}{3}\pi 9^2(30)-\frac{1}{3}\pi 6^2(20)$$

The expression $\frac{1}{3}\pi 9^2(30)$ represents the volume of the large cone, and $\frac{1}{3}\pi 6^2(20)$ is the volume of the small cone. The difference in volumes represents the volume of the truncated cone.

d. Calculate the volume of the truncated cone.

The volume of the small cone is

$$\begin{aligned}V&=\frac{1}{3}\pi 6^2(20)\\&=\frac{720}{3}\pi\end{aligned}$$

The volume of the large cone is

$$\begin{aligned}V&=\frac{1}{3}\pi 9^2(30)\\&=\frac{2{,}430}{3}\pi\end{aligned}$$

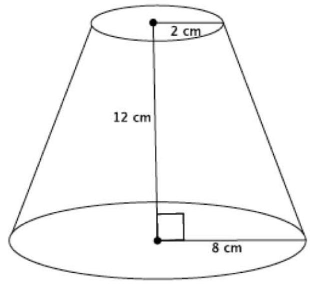
The volume of the truncated cone is

$$\begin{aligned}\frac{2{,}430}{3}\pi-\frac{720}{3}\pi&=\left(\frac{2{,}430}{3}-\frac{720}{3}\right)\pi\\&=\frac{1{,}710}{3}\pi\\&=570\pi\end{aligned}$$

The volume of the truncated cone is 570π in^3.

Students use what they know about similar triangles to determine the volume of truncated cones.

1. **Find the volume of the truncated cone.**



a. **Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.**

$$\frac{2}{8} = \frac{x}{x+12}$$

Let x represent the height of the small cone. Then $x + 12$ is the height of the large cone. The 2 represents the base radius of the small cone, and the 8 represents the base radius of the large cone.

b. **Solve your proportion to determine the height of the cone that has been removed.**

$$\begin{aligned} 2(x+12) &= 8x \\ 2x + 24 &= 8x \\ 24 &= 6x \\ 4 &= x \end{aligned}$$

c. **Show a fact about the volume of the truncated cone using an expression. Explain what each part of the expression represents.**

$$\frac{1}{3}\pi 8^2(16) - \frac{1}{3}\pi 2^2(4)$$

The expression $\frac{1}{3}\pi 8^2(16)$ represents the volume of the large cone, and $\frac{1}{3}\pi 2^2(4)$ is the volume of the small cone. The difference in volumes gives the volume of the truncated cone.

d. **Calculate the volume of the truncated cone.**

The volume of the small cone is

$$\begin{aligned} V &= \frac{1}{3}\pi 2^2(4) \\ &= \frac{16}{3}\pi \end{aligned}$$

The volume of the large cone is

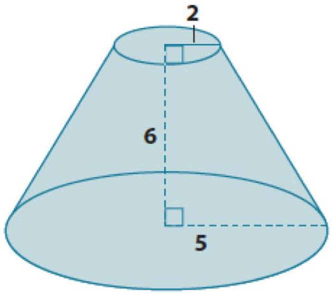
$$\begin{aligned} V &= \frac{1}{3}\pi 8^2(16) \\ &= \frac{1{,}024}{3}\pi \end{aligned}$$

The volume of the truncated cone is

$$\begin{aligned} \frac{1{,}024}{3}\pi - \frac{16}{3}\pi &= \left(\frac{1{,}024}{3} - \frac{16}{3}\right)\pi \\ &= \frac{1{,}008}{3}\pi \\ &= 336\pi \end{aligned}$$

The volume of the truncated cone is 336π cm³.

2. **Find the volume of the truncated cone.**



Let x represent the height of the small cone.

$$\frac{2}{5}=\frac{x}{x+6}$$
$$2(x+6)=5x$$
$$2x+12=5x$$
$$12=3x$$
$$4=x$$

The volume of the small cone is

$$V=\frac{1}{3}\pi 2^2(4)$$
$$=\frac{16}{3}\pi$$

The volume of the large cone is

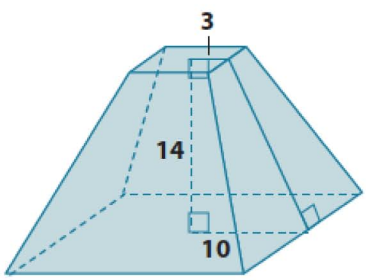
$$V=\frac{1}{3}\pi 5^2(10)$$
$$=\frac{250}{3}\pi$$

The volume of the truncated cone is

$$\frac{250}{3}\pi-\frac{16}{3}\pi=\left(\frac{250}{3}-\frac{16}{3}\right)\pi$$
$$=\frac{234}{3}\pi$$
$$=78\pi$$

The volume of the truncated cone is 78π units^3.

3. **Find the volume of the truncated pyramid with a square base.**



Let x represent the height of the small pyramid.

$$\frac{3}{10}=\frac{x}{x+14}$$
$$3(x+14)=10x$$
$$3x+42=10x$$
$$42=7x$$
$$6=x$$

The volume of the small pyramid is

$$V=\frac{1}{3}(36)(6)$$
$$=\frac{216}{3}$$

The volume of the large pyramid is

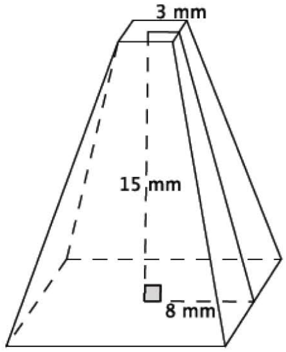
$$V=\frac{1}{3}(400)(20)$$
$$=\frac{8,000}{3}$$

The volume of the truncated pyramid is

$$\frac{8,000}{3}-\frac{216}{3}=\frac{7,784}{3}$$

The volume of the truncated pyramid is $\frac{7,784}{3}$ units^3.

4. **Find the volume of the truncated pyramid with a square base. Note: 3 mm is the distance from the center to the edge of the square at the top of the figure.**



Let x represent the height of the small pyramid.

$$\frac{3}{8} = \frac{x}{x+15}$$
$$3(x+15) = 8x$$
$$3x + 45 = 8x$$
$$45 = 5x$$
$$9 = x$$

The volume of the small pyramid is

$$V = \frac{1}{3}(36)(9) = 108$$

The volume of the large pyramid is

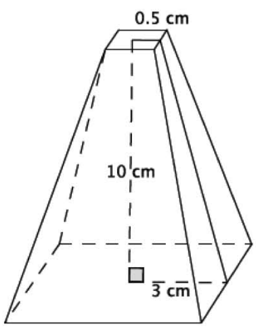
$$V = \frac{1}{3}(256)(24) = 2{,}048$$

The volume of the truncated pyramid is

$$2{,}048 - 108 = 1{,}940$$

The volume of the truncated pyramid is $1{,}940$ mm³.

5. **Find the volume of the truncated pyramid with a square base. Note: 0.5 cm is the distance from the center to the edge of the square at the top of the figure.**



Let x represent the height of the small pyramid.

$$\frac{0.5}{3} = \frac{x}{x+10}$$
$$\frac{1}{2}(x+10) = 3x$$
$$\frac{1}{2}x + 5 = 3x$$
$$5 = \frac{5}{2}x$$
$$2 = x$$

The volume of the small pyramid is

$$V = \frac{1}{3}(1)(2) = \frac{2}{3}$$

The volume of the large pyramid is

$$V = \frac{1}{3}(36)(12) = \frac{432}{3}$$

The volume of the truncated pyramid is

$$\frac{432}{3} - \frac{2}{3} = \frac{430}{3}$$

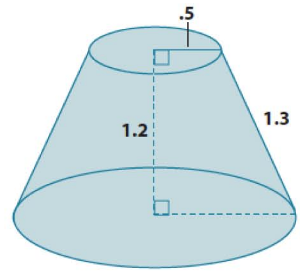
The volume of the truncated pyramid is $\frac{430}{3}$ cm³.

6. **Explain how to find the volume of a truncated cone.**

The first thing you have to do is use the ratios of corresponding sides of similar triangles to determine the height of the cone that was removed to make the truncated cone. Once you know the height of that cone, you can determine its volume. Then you can find the height of the cone (the truncated cone and the portion that was removed). Once you know both volumes you can subtract the smaller volume from the larger volume. The difference is the volume of the truncated cone.

7. **Challenge: Find the volume of the truncated cone.**

Since the height of the truncated cone is 1.2 units, we can drop a perpendicular line from the top of the cone to the bottom of the cone so that we have a right triangle with a leg length of 1.2 units and a hypotenuse of 1.3 units. Then by the Pythagorean Theorem, if b is the length of the leg of the right triangle, then



$$\begin{aligned} 1.2^2 + b^2 &= 1.3^2 \\ 1.44 + b^2 &= 1.69 \\ b^2 &= 0.25 \\ b &= 0.5 \end{aligned}$$

The part of the radius of the bottom base found by the Pythagorean Theorem is 0.5. When we add the length of the upper radius (because if you translate along the height of the truncated cone then it is equal to the remaining part of the lower base), then the radius of the lower base is 1.

Let x represent the height of the small cone:

$$\begin{aligned} \frac{0.5}{1} &= \frac{x}{x+1.2} \\ \frac{1}{2}(x+1.2) &= x \\ \frac{1}{2}x + 0.6 &= x \\ 0.6 &= \frac{1}{2}x \\ 1.2 &= x \end{aligned}$$

The volume of the small cone

$$\begin{aligned} V &= \frac{1}{3}\pi(0.5^2)(1.2) \\ &= \frac{0.3}{3}\pi \end{aligned}$$

The volume of the large cone is

$$\begin{aligned} V &= \frac{1}{3}\pi(1^2)(2.4) \\ &= \frac{2.4}{3}\pi \end{aligned}$$

The volume of the truncated cone is

$$\begin{aligned} \frac{2.4}{3}\pi - \frac{0.3}{3}\pi &= \left(\frac{2.4}{3} - \frac{0.3}{3}\right)\pi \\ &= \frac{2.1}{3}\pi \\ &= 0.7\pi \end{aligned}$$

The volume of the truncated cone is 0.7π units^3.