Name ____ Date ____________________

Converting Repeating Decimals to Fractions

1. Find the fraction equal to $0.\overline{534}$.

2. Find the fraction equal to $3.0\overline{15}$.

1. a. Let $x = 0.\overline{631}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.

   b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.

   c. Solve the equation to determine the value of x.

   d. Is your answer reasonable? Check your answer using a calculator.

2. Find the fraction equal to $3.40\overline{8}$. Check that you are correct using a calculator.

3. Find the fraction equal to $0.\overline{5923}$. Check that you are correct using a calculator.

4. Find the fraction equal to $2.3\overline{82}$. Check that you are correct using a calculator.

5. Find the fraction equal to $0.\overline{714285}$. Check that you are correct using a calculator.

6. Explain why an infinite decimal that is not a repeating decimal cannot be rational.

7. In a previous lesson we were convinced that it is acceptable to write $0.\overline{9} = 1$. Use what you learned today to show that it is true.

8. Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?

$$0.\overline{81} = \frac{81}{99} \qquad 0.\overline{4} = \frac{4}{9} \qquad 0.\overline{123} = \frac{123}{999} \qquad 0.\overline{60} = \frac{60}{99} \qquad 0.\overline{9} = 1.0$$

1. **Find the fraction equal to $0.\overline{534}$.**

Let $x = 0.\overline{534}$.

$$x = 0.\overline{534}$$
$$10^3 x = (10^3)0.\overline{534}$$
$$1,000x = 534.\overline{534}$$
$$1,000x = 534 + x$$
$$1,000x - x = 534 + x - x$$
$$999x = 534$$
$$\frac{999x}{999} = \frac{534}{999}$$
$$x = \frac{534}{999}$$
$$x = \frac{178}{333}$$

$0.\overline{534} = \frac{178}{333}$

2. **Find the fraction equal to $3.0\overline{15}$.**

Let $x = 3.0\overline{15}$

$$x = 3.0\overline{15}$$
$$10x = (10)3.0\overline{15}$$
$$10x = 30.\overline{15}$$

Let $y = 0.\overline{15}$

$$y = 0.\overline{15}$$
$$10^2 y = (10^2)0.\overline{15}$$
$$100y = 15.\overline{15}$$
$$100y = 15 + y$$
$$100y - y = 15 + y - y$$
$$99y = 15$$
$$\frac{99y}{99} = \frac{15}{99}$$
$$y = \frac{5}{33}$$

$$10x = 30.\overline{15}$$
$$10x = 30 + y$$
$$10x = 30 + \frac{5}{33}$$
$$10x = \frac{30 \times 33}{33} + \frac{5}{33}$$
$$10x = \frac{30 \times 33 + 5}{33}$$
$$10x = \frac{995}{33}$$
$$\frac{1}{10}(10x) = \frac{1}{10}\left(\frac{995}{33}\right)$$
$$x = \frac{995}{330}$$
$$x = \frac{199}{66}$$

$3.0\overline{15} = \frac{199}{66}$

1. a. Let $x = 0.\overline{631}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.

When we multiply both sides of the equation by 10^3, on the right side we will have $631.631631\ldots$. This is helpful because we will be able to subtract the repeating decimal from both sides by subtracting x.

b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.

$$x = 0.\overline{631}$$
$$10^3 x = (10^3)0.\overline{631}$$
$$1{,}000x = 631.\overline{631}$$
$$1{,}000x = 631 + 0.631631\ldots$$
$$1{,}000x = 631 + x$$

Since we let $x = 0.\overline{631}$, we can substitute the repeating decimal $0.631631\ldots$ with x.

c. Solve the equation to determine the value of x.

$$1{,}000x - x = 631 + x - x$$
$$999x = 631$$
$$\frac{999x}{999} = \frac{631}{999}$$
$$x = \frac{631}{999}$$

d. Is your answer reasonable? Check your answer using a calculator.

Yes, my answer is reasonable and correct. It is reasonable because the denominator cannot be expressed as a product of 2's and 5's; therefore, I know that the fraction must represent an infinite decimal. Also the number 0.631 is closer to 0.5 than 1, and the fraction is also closer to $\frac{1}{2}$ than 1. It is correct because the division $\frac{631}{999}$ using the calculator is $0.631631\ldots$.

2. Find the fraction equal to $3.40\overline{8}$. Check that you are correct using a calculator.

Let $x = 3.40\overline{8}$

$$x = 3.40\overline{8}$$
$$10^2 x = (10^2)3.40\overline{8}$$
$$100x = 340.\overline{8}$$

Let $y = 0.\overline{8}$

$$y = 0.\overline{8}$$
$$10y = 10(0.\overline{8})$$
$$10y = 8.\overline{8}$$
$$10y = 8 + y$$
$$10y - y = 8 + y - y$$
$$9y = 8$$
$$\frac{9y}{9} = \frac{8}{9}$$
$$y = \frac{8}{9}$$

$$100x = 340.\overline{8}$$
$$100x = 340 + y$$
$$100x = 340 + \frac{8}{9}$$
$$100x = \frac{340 \times 9}{9} + \frac{8}{9}$$
$$100x = \frac{340 \times 9 + 8}{9}$$
$$100x = \frac{3{,}068}{9}$$
$$\left(\frac{1}{100}\right)100x = \left(\frac{1}{100}\right)\frac{3{,}068}{9}$$
$$x = \frac{3{,}068}{900}$$
$$x = \frac{767}{225}$$

$$3.40\overline{8} = \frac{767}{225}$$

3. **Find the fraction equal to $0.\overline{5923}$. Check that you are correct using a calculator.**

Let $x = 0.\overline{5923}$

$$\begin{aligned} x &= 0.\overline{5923} \\ 10^4x &= (10^4)0.\overline{5923} \\ 10{,}000x &= 5{,}923.\overline{5923} \\ 10{,}000x &= 5{,}923 + x \\ 10{,}000x - x &= 5{,}923 + x - x \\ 9{,}999x &= 5{,}923 \\ \frac{9{,}999x}{9{,}999} &= \frac{5{,}923}{9{,}999} \\ x &= \frac{5{,}923}{9{,}999} \end{aligned}$$

4. **Find the fraction equal to $2.3\overline{82}$. Check that you are correct using a calculator.**

Let $x = 2.3\overline{82}$

$$\begin{aligned} x &= 2.3\overline{82} \\ 10x &= (10)2.3\overline{82} \\ 10x &= 23.\overline{82} \end{aligned}$$

Let $y = 0.\overline{82}$

$$\begin{aligned} y &= 0.\overline{82} \\ 10^2y &= (10^2)0.\overline{82} \\ 100y &= 82.\overline{82} \\ 100y &= 82 + y \\ 100y - y &= 82 + y - y \\ 99y &= 82 \\ \frac{99y}{99} &= \frac{82}{99} \\ y &= \frac{82}{99} \end{aligned}$$

$$\begin{aligned} 10x &= 23.\overline{82} \\ 10x &= 23 + \frac{82}{99} \\ 10x &= \frac{23 \times 99}{99} + \frac{82}{99} \\ 10x &= \frac{23 \times 99 + 82}{99} \\ 10x &= \frac{2{,}359}{99} \\ \left(\frac{1}{10}\right)10x &= \left(\frac{1}{10}\right)\frac{2{,}359}{99} \\ x &= \frac{2{,}359}{990} \end{aligned}$$

$$2.3\overline{82} = \frac{2{,}359}{990}$$

5. **Find the fraction equal to $0.\overline{714285}$. Check that you are correct using a calculator.**

Let $x = 0.\overline{714285}$.

$$\begin{aligned} x &= 0.\overline{714285} \\ 10^6x &= (10^6)0.\overline{714285} \\ 1{,}000{,}000x &= 714{,}825.\overline{714285} \\ 1{,}000{,}000x &= 714{,}285 + x \\ 1{,}000{,}000x - x &= 714{,}285 + x - x \\ 999{,}999x &= 714{,}285 \\ \frac{999{,}999x}{999{,}999} &= \frac{714{,}285}{999{,}999} \\ x &= \frac{714{,}285}{999{,}999} \\ x &= \frac{5}{7} \end{aligned}$$

6. **Explain why an infinite decimal that is not a repeating decimal cannot be rational.**

Infinite decimals that do repeat can be expressed as a fraction and are therefore rational. The method we learned today to write a repeating decimal as a rational number cannot be applied to infinite decimals that do not repeat. The method requires that we let x represent the repeating part of the decimal. If the number has a decimal expansion that does not repeat, we cannot express the number as a fraction, i.e., a rational number.

7. **In a previous lesson we were convinced that it is acceptable to write $0.\overline{9} = 1$. Use what you learned today to show that it is true.**

Let $x = 0.\overline{9}$

$$x = 0.\overline{9}$$
$$10x = (10)0.\overline{9}$$
$$10x = 9.\overline{9}$$
$$10x = 9 + x$$
$$10x - x = 9 + x - x$$
$$9x = 9$$
$$\frac{9x}{9} = \frac{9}{9}$$
$$x = \frac{9}{9}$$
$$x = 1$$

8. **Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?**

$$0.\overline{81} = \frac{81}{99} \qquad 0.\overline{4} = \frac{4}{9} \qquad 0.\overline{123} = \frac{123}{999} \qquad 0.\overline{60} = \frac{60}{99} \qquad 0.\overline{9} = 1.0$$

In each case, the fraction that represents the infinite decimal has a numerator that is exactly the repeating part of the decimal and a denominator comprised of 9's. Specifically, the denominator has the same number of digits of 9's as the number of digits that repeat. For example, $0.\overline{81}$ has two repeating decimal digits, so the denominator has two 9's. Since we know that $0.\overline{9} = 1$, we can make the assumption that repeating 9's, like 99 could be expressed as 100, meaning that the fraction $\frac{81}{99}$ is almost $\frac{81}{100}$, which would then be expressed as 0.81.