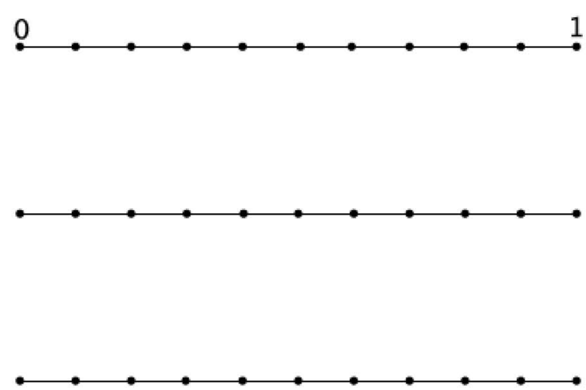
Name ____ Date ________________

Infinite Decimals

1. a. Write the expanded form of the decimal 0.829 using powers of 10.

   b. Show on the number line the representation of the decimal 0.829.



   c. Is the decimal finite or infinite? How do you know?

Convert each fraction to a finite decimal. If the fraction cannot be written as a finite decimal, then state how you know. Show your steps, but use a calculator for the multiplications.

1. $\frac{2}{32}$

2. $\frac{99}{125}$

   a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{99}{125}$.

   b. Find the decimal representation of $\frac{99}{125}$. Explain why your answer is reasonable.

3. $\frac{15}{128}$

4. $\frac{8}{15}$

5. $\frac{3}{28}$

6. $\frac{13}{400}$

7. $\frac{5}{64}$

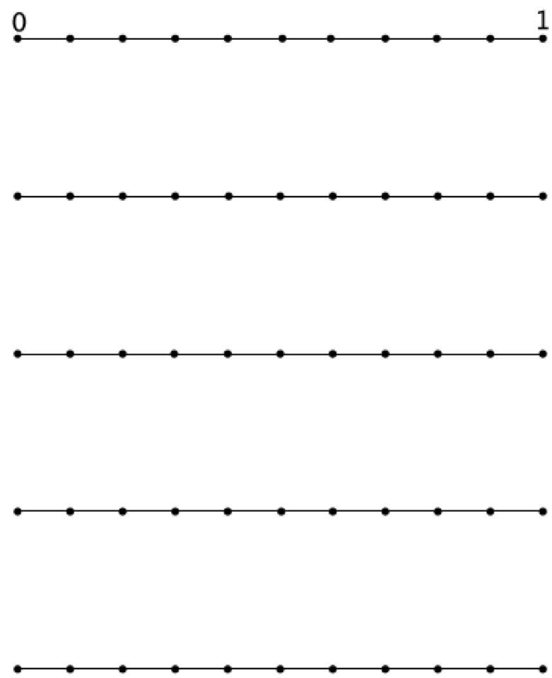
8. $\frac{15}{35}$

9. $\frac{199}{250}$

10. $\frac{219}{625}$

2. a. Write the expanded form of the decimal $0.55555\ldots$ using powers of 10.

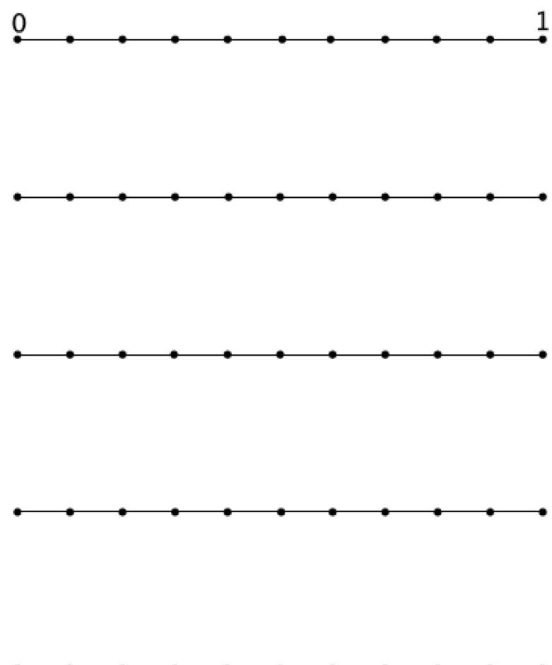
b. Show on the number line the representation of the decimal $0.555555\ldots$



c. Is the decimal finite or infinite? How do you know?

3. a. Write the expanded form of the decimal $0.\overline{573}$ using powers of 10.

b. Show on the number line the representation of the decimal $0.\overline{573}$.

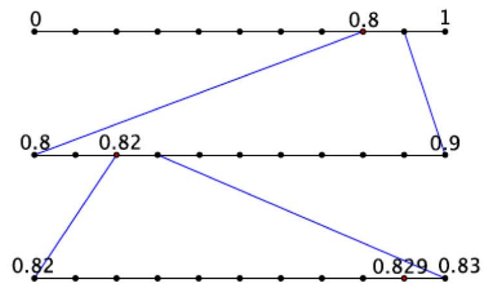


c. Is the decimal finite or infinite? How do you know?

1. a. Write the expanded form of the decimal 0.829 using powers of 10.

$$0.829 = \frac{8}{10} + \frac{2}{10^2} + \frac{9}{10^3}$$

b. Show on the number line the representation of the decimal 0.829.



c. Is the decimal finite or infinite? How do you know?

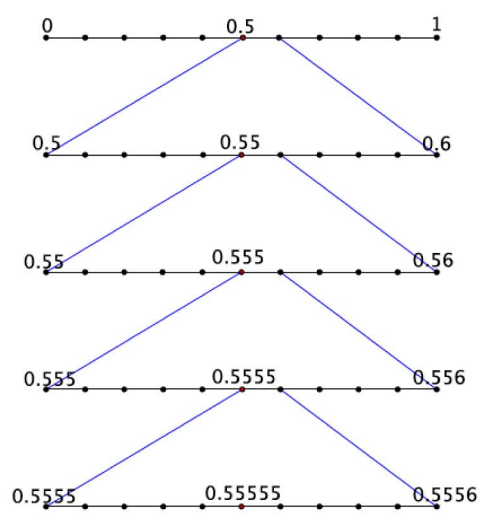
The decimal 0.829 is finite because it can be completely represented by a finite number of steps.

2. a. Write the expanded form of the decimal $0.55555\ldots$ using powers of 10.

$$0.55555\ldots = \frac{5}{10} + \frac{5}{10^2} + \frac{5}{10^3} + \frac{5}{10^4} + \frac{5}{10^5} + \frac{5}{10^6} + \cdots$$

and so on.

b. Show on the number line the representation of the decimal $0.555555\ldots$



c. Is the decimal finite or infinite? How do you know?

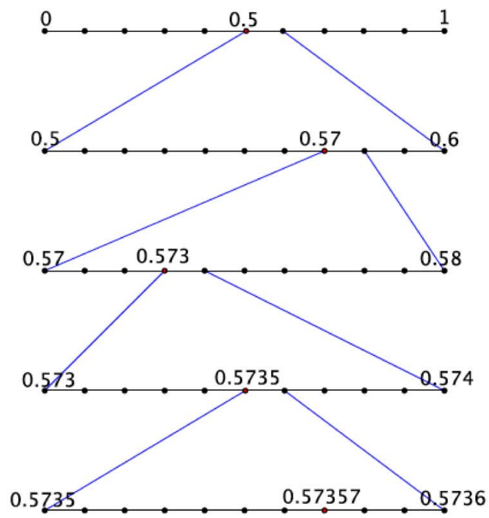
The decimal $0.555555\ldots$ is infinite because it cannot be represented by a finite number of steps. Because the number 5 continues to repeat, there will be an infinite number of steps.

3. a. Write the expanded form of the decimal $0.\overline{573}$ using powers of 10.

$$0.\overline{573} = \frac{5}{10} + \frac{7}{10^2} + \frac{3}{10^3} + \frac{5}{10^4} + \frac{7}{10^5} + \frac{3}{10^6} + \cdots$$

and so on.

b. Describe the sequence that would represent the decimal $0.\overline{573}$.



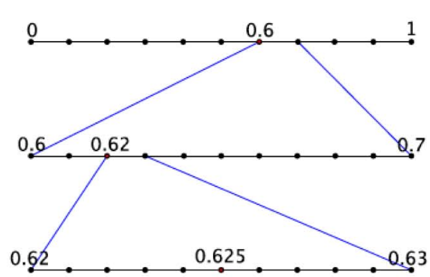
c. Is the decimal finite or infinite? How do you know?

The decimal $0.\overline{573}$ is infinite because it cannot be represented by a finite number of steps. Because the digits 5, 7, and 3 continue to repeat, there will be an infinite number of steps.

1. a. Write the expanded form of the decimal 0.625 using powers of 10.

$$0.625 = \frac{6}{10} + \frac{2}{10^2} + \frac{5}{10^3}$$

b. Show on the number line the representation of the decimal 0.625.



c. **Is the decimal finite or infinite? How do you know?**

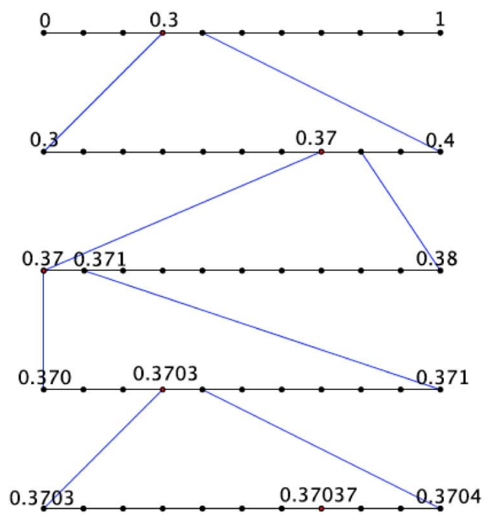
The decimal 0.625 is finite because it can be completely represented by a finite number of steps in the sequence.

2. a. **Write the expanded form of the decimal $0.\overline{370}$ using powers of 10.**

$$0.\overline{370} = \frac{3}{10} + \frac{7}{10^2} + \frac{0}{10^3} + \frac{3}{10^4} + \frac{7}{10^5} + \frac{0}{10^6} + \cdots$$

and so on.

b. **Show on the number line the representation of the decimal $0.370370\ldots$**



c. **Is the decimal finite or infinite? How do you know?**

The decimal $0.\overline{370}$ is infinite because it cannot be represented by a finite number of steps. Because the digits 3, 7, and 0 continue to repeat, there will be an infinite number of steps in the sequence.

3. **Which is a more accurate representation of the number $\frac{2}{3}$: 0.6666 or $0.\overline{6}$? Explain. Which would you prefer to compute with?**

The number $\frac{2}{3}$ is more accurately represented by the decimal $0.\overline{6}$ compared to 0.6666. The long division algorithm with $\frac{2}{3}$ shows that the digit 6 repeats. Then the expanded form of the decimal $0.\overline{6} = \frac{6}{10} + \frac{6}{10^2} + \frac{6}{10^3} + \frac{6}{10^4} + \frac{6}{10^5} + \frac{6}{10^6} + \cdots$, and so on, where the number $0.6666 = \frac{6}{10} + \frac{6}{10^2} + \frac{6}{10^3} + \frac{6}{10^4}$. For this reason, $0.\overline{6}$ is more accurate. For computations, I would prefer to use 0.6666. My answer would be less precise, but at least I'd be able to compute with it. When attempting to compute with an infinite number, you would never finish writing it, thus you could never compute with it.

4. **Explain why we shorten infinite decimals to finite decimals to perform operations. Explain the effect of shortening an infinite decimal on our answers.**

We often shorten infinite decimals to finite decimals to perform operations because it would be impossible to represent an infinite decimal precisely because the sequence that describes infinite decimals has an infinite number of steps. Our answers are less precise; however, they are not that much less precise because with each additional digit in the sequence we include, we are adding a very small amount to the value of the number. The more decimals we include, the closer the value we add approaches zero. Therefore, it does not make that much of a difference with respect to our answer.

5. **A classmate missed the discussion about why $0.\overline{9} = 1$. Convince your classmate that this equality is true.**

When you consider the infinite sequence of steps that represents the decimal $0.9999999\ldots$, it is clear that the value we add with each step is an increasingly smaller value, so it makes sense to write that $0.\overline{9} = 1$. As we increase the number of steps in the sequence, we are adding smaller and smaller values to the number. Consider the 12^{th} step: 0.999999999999. The value added to the number is just 0.000000000009, which is a very small amount. The more steps that we include, the closer that value is to zero. Which means that with each new step, the number $0.\overline{9}$ gets closer and closer to 1. Since this process is infinite, the number $0.\overline{9} = 1$.

6. **Explain why $0.3333 < 0.33333$.**

The number $0.3333 = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4}$, and the number $0.33333 = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \frac{3}{10^5}$. That means that 0.33333 is exactly $\frac{3}{10^5}$ larger than 0.3333. If we examined the numbers on the number line, 0.33333 is to the right of 0.3333 meaning that it is larger than 0.3333.