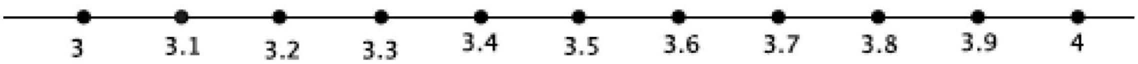
Name ____ Date ___________________

Comparison of Irrational Numbers

Place the following numbers at their approximate location on the number line: $\sqrt{12}$, $\sqrt{16}$, $\frac{20}{6}$, $3.\overline{53}$, $\sqrt[3]{27}$.



1. Which number is smaller, $\sqrt[3]{343}$ or $\sqrt{48}$? Explain.

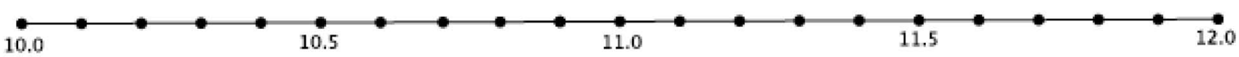
2. Which number is smaller, $\sqrt{100}$ or $\sqrt[3]{1000}$? Explain.

3. Which number is larger, $\sqrt{87}$ or $\frac{929}{99}$? Explain.

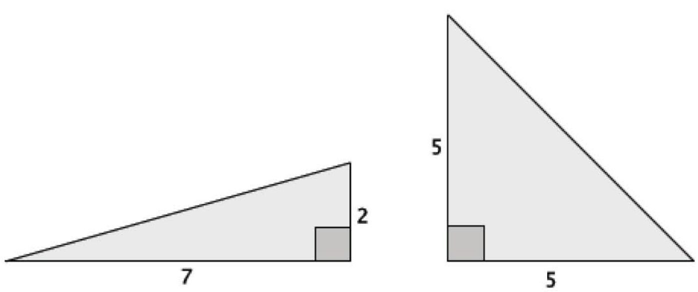
4. Which number is larger, $\frac{9}{13}$ or $0.\overline{692}$? Explain.

5. Which number is larger, 9.1 or $\sqrt{82}$? Explain.

6. Place the following numbers at their approximate location on the number line: $\sqrt{144}$, $\sqrt[3]{1000}$, $\sqrt{130}$, $\sqrt{110}$, $\sqrt{120}$, $\sqrt{115}$, $\sqrt{133}$. Explain how you knew where to place the numbers.



7. Which of the two right triangles shown below, measured in units, has the longer hypotenuse? Approximately how much longer is it?



Place the following numbers at their approximate location on the number line: $\sqrt{12}, \sqrt{16}, \frac{20}{6}, 3.\overline{53}, \sqrt[3]{27}$.

Note that students may have used long division or the method of rational approximation to determine the decimal expansion of the fraction.

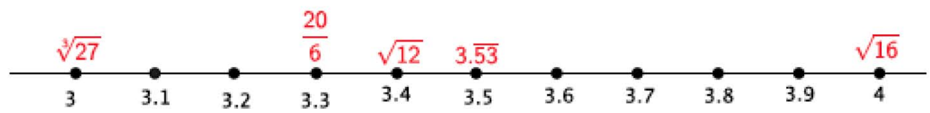
The number $\sqrt{12}$ is between 3.4 and 3.5, since $3.4^2 < (\sqrt{12})^2 < 3.5^2$.

The number $\sqrt{16} = \sqrt{4^2} = 4$.

The number $\frac{20}{6}$ is equal to $3.\overline{3}$.

The number $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$.

Solutions in red:



1. Which number is smaller, $\sqrt[3]{343}$ or $\sqrt{48}$? Explain.

$$\sqrt[3]{343} = \sqrt[3]{7^3} = 7$$

The number $\sqrt{48}$ is between 6 and 7, but definitely less than 7. Therefore, $\sqrt{48} < \sqrt[3]{343}$ and $\sqrt{48}$ is smaller.

2. Which number is smaller, $\sqrt{100}$ or $\sqrt[3]{1000}$? Explain.

$$\sqrt{100} = \sqrt{10^2} = 10$$

$$\sqrt[3]{1,000} = \sqrt[3]{10^3} = 10$$

The numbers $\sqrt{100}$ and $\sqrt[3]{1,000}$ are equal because both are equal to 10.

3. Which number is larger, $\sqrt{87}$ or $\frac{929}{99}$? Explain.

Note that students may have used long division or the method of rational approximation to determine the decimal expansion of the fraction.

The number $\frac{929}{99}$ is equal to $9.\overline{38}$.

The number $\sqrt{87}$ is between 9 and 10 because $9^2 < \left(\sqrt{87}\right)^2 < 10^2$. The number $\sqrt{87}$ is between 9.3 and 9.4 because $9.3^2 < \left(\sqrt{87}\right)^2 < 9.4^2$. The number $\sqrt{87}$ is between 9.32 and 9.33 because $9.32^2 < \left(\sqrt{87}\right)^2 < 9.33^2$. The approximate decimal value of $\sqrt{87}$ is 9.32 …. Since $9.32 < 9.\overline{38}$, then $\sqrt{87} < \frac{929}{99}$; therefore, the fraction $\frac{929}{99}$ is larger.

4. **Which number is larger, $\frac{9}{13}$ or $0.\overline{692}$? Explain.**

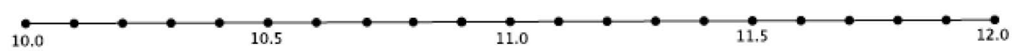
Note that students may have used long division or the method of rational approximation to determine the decimal expansion of the fraction.

The number $\frac{9}{13}$ is equal to $0.\overline{692307}$. Since $0.692307\ldots < 0.692692\ldots$, then $\frac{9}{13} < 0.\overline{692}$; therefore, the decimal $0.\overline{692}$ is larger.

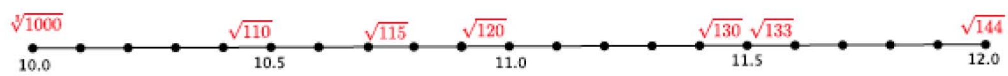
5. **Which number is larger, 9.1 or $\sqrt{82}$? Explain.**

The number $\sqrt{82}$ is between 9 and 10 because $9^2 < \left(\sqrt{82}\right)^2 < 10^2$. The number $\sqrt{82}$ is between 9.0 and 9.1 because $9.0^2 < \left(\sqrt{82}\right)^2 < 9.1^2$. Since $\sqrt{82} < 9.1$, then the number 9.1 is larger than the number $\sqrt{82}$.

6. **Place the following numbers at their approximate location on the number line: $\sqrt{144}$, $\sqrt[3]{1000}$, $\sqrt{130}$, $\sqrt{110}$, $\sqrt{120}$, $\sqrt{115}$, $\sqrt{133}$. Explain how you knew where to place the numbers.**



Solutions shown in red



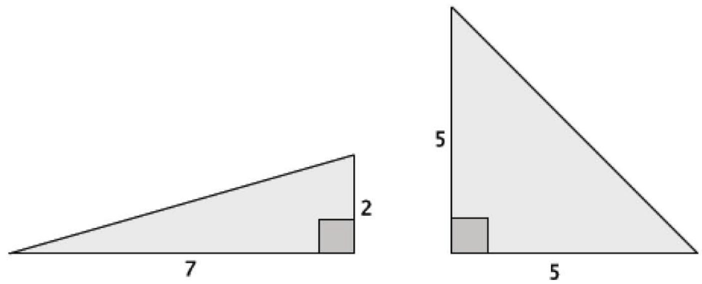
The number $\sqrt{144} = \sqrt{12^2} = 12$.

The number $\sqrt[3]{1000} = \sqrt[3]{10^3} = 10$.

The numbers $\sqrt{110}$, $\sqrt{115}$, and $\sqrt{120}$ are all between 10 and 11 because when squared, their value falls between 10^2 and 11^2. The number $\sqrt{110}$ is between 10.4 and 10.5 because $10.4^2 < \left(\sqrt{110}\right)^2 < 10.5^2$. The number $\sqrt{115}$ is between 10.7 and 10.8 because $10.7^2 < \left(\sqrt{115}\right)^2 < 10.8^2$. The number $\sqrt{120}$ is between 10.9 and 11 because $10.9^2 < \left(\sqrt{120}\right)^2 < 11^2$.

The numbers $\sqrt{130}$ and $\sqrt{133}$ are between 11 and 12 because when squared, their value falls between 11^2 and 12^2. The number $\sqrt{130}$ is between 11.4 and 11.5 because $11.4^2 < \left(\sqrt{130}\right)^2 < 11.5^2$. The number $\sqrt{133}$ is between 11.5 and 11.6 because $11.5^2 < \left(\sqrt{133}\right)^2 < 11.6^2$.

7. **Which of the two right triangles shown below, measured in units, has the longer hypotenuse? Approximately how much longer is it?**



Let x represent the hypotenuse of the triangle on the left.

$$\begin{aligned} 7^2 + 2^2 &= x^2 \\ 49 + 4 &= x^2 \\ 53 &= x^2 \\ \sqrt{53} &= \sqrt{x^2} \\ \sqrt{53} &= x \end{aligned}$$

The number $\sqrt{53}$ is between 7 and 8 because $7^2 < \left(\sqrt{53}\right)^2 < 8^2$. The number $\sqrt{53}$ is between 7.2 and 7.3 because $7.2^2 < \left(\sqrt{53}\right)^2 < 7.3^2$. The number $\sqrt{53}$ is between 7.28 and 7.29 because $7.28^2 < \left(\sqrt{53}\right)^2 < 7.29^2$. The approximate decimal value of $\sqrt{53}$ is 7.28 ….

Let y represent the hypotenuse of the triangle on the right.

$$\begin{aligned} 5^2 + 5^2 &= y^2 \\ 25 + 25 &= y^2 \\ 50 &= y^2 \\ \sqrt{50} &= \sqrt{y^2} \\ \sqrt{50} &= y \end{aligned}$$

The number $\sqrt{50}$ is between 7 and 8 because $7^2 < \left(\sqrt{50}\right)^2 < 8^2$. The number $\sqrt{50}$ is between 7.0 and 7.1 because $7.0^2 < \left(\sqrt{50}\right)^2 < 7.1^2$. The number $\sqrt{50}$ is between 7.07 and 7.08 because $7.07^2 < \left(\sqrt{50}\right)^2 < 7.08^2$. The approximate decimal value of $\sqrt{50}$ is 7.07 ….

The triangle on the left has the longer hypotenuse. It is approximately 0.21 units longer than the hypotenuse of the triangle on the right.

Note: Based on their experience, some students may reason that $\sqrt{50} < \sqrt{53}$. To answer completely, students must determine the decimal expansion to approximate how much longer one hypotenuse is than the other.