Name ____ Date ________________

The Decimal Expansion of Some Irrational Numbers

1. Determine the 3 decimal digit approximation of the number $\sqrt{17}$.

2. Classify the following numbers as rational or irrational, and explain how you know.

$$\frac{3}{5}, \quad 0.73737373\ldots, \quad \sqrt{31}$$

1. Use the method of rational approximation to determine the decimal expansion of $\sqrt{84}$. Determine which interval of hundredths it would lie in.

2. Get a 3 decimal digit approximation of the number $\sqrt{34}$.

3. Write the decimal expansion of $\sqrt{47}$ to at least 2 decimal digits.

4. Write the decimal expansion of $\sqrt{46}$ to at least 2 decimal digits.

5. Explain how to improve the accuracy of decimal expansion of an irrational number.

6. Is the number $\sqrt{125}$ rational or irrational? Explain.

7. Is the number $0.646464646\ldots$ rational or irrational? Explain.

8. Is the number $3.741657387\ldots$ rational or irrational? Explain.

9. Is the number $\sqrt{99}$ rational or irrational? Explain.

10. Challenge: Get a 2 decimal digit approximation of the number $\sqrt[3]{9}$.

1. **Determine the 3 decimal digit approximation of the number $\sqrt{17}$.**

The number $\sqrt{17}$ is between integers 4 and 5 because $4^2 < \left(\sqrt{17}\right)^2 < 5^2$. Since $\sqrt{17}$ is closer to 4, I will start checking the tenths intervals closer to 4. $\sqrt{17}$ is between 4.1 and 4.2 since $4.1^2 = 16.81$ and $4.2^2 = 17.64$. Checking the hundredths interval, $\sqrt{17}$ is between 4.12 and 4.13 since $4.12^2 = 16.9744$ and $4.13^2 = 17.0569$. Checking the thousandths interval, $\sqrt{17}$ is between 4.123 and 4.124 since $4.123^2 = 16.999129$ and $4.124^2 = 17.007376$. Since 17 is closer to 4.123^2 than 4.124^2, then the three decimal approximation is approximately 4.123.

2. **Classify the following numbers as rational or irrational, and explain how you know.**

$$\frac{3}{5}, \quad 0.73737373\ldots, \quad \sqrt{31}$$

The number $\frac{3}{5}$, by definition, is rational because it is a ratio of integers. The number $0.73737373\ldots$ is rational because it has a repeat block. For that reason, the number can be expressed as a fraction. The number $\sqrt{31}$ is irrational because it has a decimal expansion that can only be approximated by rational numbers. That is, the number is not equal to a rational number; therefore, it is irrational.

1. **Use the method of rational approximation to determine the decimal expansion of $\sqrt{84}$. Determine which interval of hundredths it would lie in.**

The number $\sqrt{84}$ is between 9 and 10 but closer to 9. Looking at the interval of tenths, beginning with 9.0 to 9.1, the number $\sqrt{84}$ lies between 9.1 and 9.2 because $9.1^2 = 82.81$ and $9.2^2 = 84.64$ but is closer to 9.2. In the interval of hundredths, the number $\sqrt{84}$ lies between 9.16 and 9.17 because $9.16^2 = 83.9056$ and $9.17^2 = 84.0889$.

2. **Get a 3 decimal digit approximation of the number $\sqrt{34}$.**

The number $\sqrt{34}$ is between 5 and 6 but closer to 6. Looking at the interval of tenths, beginning with 5.9 to 6.0, the number $\sqrt{34}$ lies between 5.8 and 5.9 because $5.8^2 = 33.64$ and $5.9^2 = 34.81$ and is closer to 5.8. In the interval of hundredths, the number $\sqrt{34}$ lies between 5.83 and 5.84 because $5.83^2 = 33.9889$ and $5.84^2 = 34.1056$ and is closer to 5.83. In the interval of thousandths, the number $\sqrt{34}$ lies between 5.830 and 5.831 because $5.830^2 = 33.9889$ and $5.831^2 = 34.000561$ but is closer to 5.831. Since 34 is closer to 5.831^2 than 5.830^2, then the 3 decimal digit approximation of the number is approximately 5.831.

3. **Write the decimal expansion of $\sqrt{47}$ to at least 2 decimal digits.**

The number $\sqrt{47}$ is between 6 and 7 but closer to 7 because $6^2 < \left(\sqrt{47}\right)^2 < 7^2$. In the interval of tenths, the number $\sqrt{47}$ is between 6.8 and 6.9 because $6.8^2 = 46.24$ and $6.9^2 = 47.61$. In the interval of hundredths, the number $\sqrt{47}$ is between 6.85 and 6.86 because $6.85^2 = 46.9225$ and $6.86^2 = 47.0596$. Therefore, to 2 decimal digits, the number $\sqrt{47}$ is approximately 6.85 but when rounded will be approximately 6.86 because $\sqrt{47}$ is closer to 6.86 but not quite 6.86.

4. **Write the decimal expansion of $\sqrt{46}$ to at least 2 decimal digits.**

The number $\sqrt{46}$ is between integers 6 and 7 because $6^2 < \left(\sqrt{46}\right)^2 < 7^2$. Since $\sqrt{46}$ is closer to 7, I will start checking the tenths intervals between 6.9 and 7. $\sqrt{46}$ is between 6.7 and 6.8 since $6.7^2 = 44.89$ and $6.8^2 = 46.24$. Checking the hundredths interval, $\sqrt{46}$ is between 6.78 and 6.79 since $6.78^2 = 45.9684$ and $6.79^2 = 46.1041$. Since 46 is closer to 6.78^2 than 6.79^2, then the two decimal approximation is 6.78.

5. **Explain how to improve the accuracy of decimal expansion of an irrational number.**

In order to improve the accuracy of the decimal expansion of an irrational number, you must examine increasingly smaller increments on the number line. Specifically, increments of decreasing powers of 10. The Basic Inequality allows us to determine which interval a number will be between. We begin by determining which two integers the number lies between and then decrease the power of 10 to look at the interval of tenths. Again using the Basic Inequality, we can narrow down the approximation to a specific interval of tenths. Then we look at the interval of hundredths and use the Basic Inequality to determine which interval of hundredths the number would lie between. Then we examine the interval of thousandths. Again the Basic Inequality allows us to narrow down the approximation to thousandths. The more intervals that are examined, the more accurate the decimal expansion of an irrational number will be.

6. **Is the number $\sqrt{125}$ rational or irrational? Explain.**

The number $\sqrt{125}$ is an irrational number because it has a decimal expansion that is infinite and does not repeat. That is, the number $\sqrt{125}$ cannot be expressed as a rational number; therefore, it is irrational.

7. **Is the number $0.646464646\ldots$ rational or irrational? Explain.**

The number $0.646464646\ldots = \frac{64}{99}$; therefore, it is a rational number. Not only is the number $\frac{64}{99}$ a quotient of integers, but its decimal expansion is infinite with a repeating block of digits.

8. **Is the number $3.741657387\ldots$ rational or irrational? Explain.**

The number $3.741657387\ldots$ is an irrational number because it has a decimal expansion that is infinite and does not repeat. That is, the number $3.741657387\ldots$ cannot be expressed as a rational number; therefore, it is irrational.

9. **Is the number $\sqrt{99}$ rational or irrational? Explain.**

The number $\sqrt{99}$ is an irrational number because it has a decimal expansion that is infinite and does not repeat. That is, the number $\sqrt{99}$ cannot be expressed as a rational number; therefore, it is irrational.

10. **Challenge: Get a 2 decimal digit approximation of the number $\sqrt[3]{9}$.**

The number $\sqrt[3]{9}$ is between integers 2 and 3 because $2^3 < \left(\sqrt[3]{9}\right)^3 < 3^3$. Since $\sqrt[3]{9}$ is closer to 2, I will start checking the tenths intervals between 2 and 3. $\sqrt[3]{9}$ is between 2 and 2.1 since $2^3 = 8$ and $2.1^3 = 9.261$. Checking the hundredths interval, $\sqrt[3]{9}$ is between 2.08 and 2.09 since $2.08^3 = 8.998912$ and $2.09^3 = 9.129329$. Since 9 is closer to 2.08^3 than 2.09^3, the two decimal approximation is 2.08.