Name ____ Date ____________________

Decimal Expansions of Fractions, Part 2

Use rational approximation to determine the decimal expansion of $\frac{41}{6}$.

1. Explain why the tenths digit of $\frac{3}{11}$ is 2, using rational approximation.
2. Use rational approximation to determine the decimal expansion of $\frac{25}{9}$.
3. Use rational approximation to determine the decimal expansion of $\frac{11}{41}$ to at least 5 digits.
4. Use rational approximation to determine which number is larger, $\sqrt{10}$ or $\frac{28}{9}$.
5. Sam says that $\frac{7}{11} = 0.63$, and Jaylen says that $\frac{7}{11} = 0.636$. Who is correct? Why?

Use rational approximation to determine the decimal expansion of $\frac{41}{6}$.

$$\frac{41}{6} = \frac{36}{36} + \frac{5}{6}$$
$$= 6 + \frac{5}{6}$$

The ones digit is 6. In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{5}{6} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{50}{6} < m + 1$$

$$\frac{50}{6} = \frac{48}{6} + \frac{2}{6}$$
$$= 8 + \frac{1}{3}.$$

The tenths digit is 8. The difference between $\frac{5}{6}$ and $\frac{8}{10}$ is

$$\frac{5}{6} - \frac{8}{10} = \frac{1}{30}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{1}{30} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{10}{3} < m + 1$$

$$\frac{10}{3} = \frac{9}{3} + \frac{1}{3}$$
$$= 3 + \frac{1}{3}.$$

The hundredths digit is 3. Again, we see the fraction $\frac{1}{3}$, which means the next decimal digit will be 3, as it was in the hundredths place. This means we will again see the fraction $\frac{1}{3}$, meaning we will have another digit of 3. Therefore, the decimal expansion of $\frac{41}{6}$ is $6.8333\ldots$.

1. **Explain why the tenths digit of $\frac{3}{11}$ is 2, using rational approximation.**

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{3}{11} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{30}{11} < m+1$$
$$\frac{30}{11} = \frac{22}{11} + \frac{8}{11}$$
$$= 2 + \frac{8}{11}.$$

In looking at the interval of tenths, we see that the number $\frac{3}{11}$ must be between $\frac{2}{10}$ and $\frac{3}{10}$ because $\frac{2}{10} < \frac{3}{11} < \frac{3}{10}$. For this reason, the tenths digit of the decimal expansion of $\frac{3}{11}$ must be 2.

2. **Use rational approximation to determine the decimal expansion of $\frac{25}{9}$.**

$$\frac{25}{9} = \frac{18}{9} + \frac{7}{9}$$
$$= 2 + \frac{7}{9}$$

The ones digit is 2. In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{7}{9} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{70}{9} < m+1$$
$$\frac{70}{9} = \frac{63}{9} + \frac{7}{9}$$
$$= 7 + \frac{7}{9}.$$

The tenths digit is 7. The difference between $\frac{7}{9}$ and $\frac{7}{10}$ is

$$\frac{7}{9} - \frac{7}{10} = \frac{7}{90}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{7}{90} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{70}{9} < m+1.$$

But we already know that $\frac{70}{9} = 7 + \frac{7}{9}$; therefore, the hundredths digit is 7. Because we keep getting $\frac{7}{9}$, we can assume the digit of 7 will continue to repeat. Therefore, the decimal expansion of $\frac{25}{9} = 2.777\ldots$.

3. **Use rational approximation to determine the decimal expansion of $\frac{11}{41}$ to at least 5 digits.**

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{11}{41} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{110}{41} < m+1$$

$$\frac{110}{41} = \frac{82}{41} + \frac{28}{41} = 2 + \frac{28}{41}.$$

The tenths digit is 2. The difference between $\frac{11}{41}$ and $\frac{2}{10}$ is

$$\frac{11}{41} - \frac{2}{10} = \frac{28}{410}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{28}{410} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{280}{41} < m+1$$

$$\frac{280}{41} = \frac{246}{41} + \frac{34}{41} = 6 + \frac{34}{41}.$$

The hundredths digit is 6. The difference between $\frac{11}{41}$ and $\left(\frac{2}{10} + \frac{6}{100}\right)$ is

$$\frac{11}{41} - \left(\frac{2}{10} + \frac{6}{100}\right) = \frac{11}{41} - \frac{26}{100} = \frac{34}{4100}.$$

In the interval of thousandths, we are looking for integers m and $m+1$ so that

$$\frac{m}{1000} < \frac{34}{4100} < \frac{m+1}{1000},$$

which is the same as

$$m < \frac{340}{41} < m+1$$

$$\frac{340}{41} = \frac{328}{41} + \frac{12}{41} = 8 + \frac{12}{41}.$$

The thousandths digit is 8. The difference between $\frac{11}{41}$ and $\left(\frac{2}{10} + \frac{6}{100} + \frac{8}{1000}\right)$ is

$$\frac{11}{41} - \left(\frac{2}{10} + \frac{6}{100} + \frac{8}{1000}\right) = \frac{11}{41} - \frac{268}{1000} = \frac{12}{41000}.$$

In the interval of ten-thousandths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10000} < \frac{12}{41000} < \frac{m+1}{10000},$$

which is the same as

$$m < \frac{120}{41} < m+1$$

$$\frac{120}{41} = \frac{82}{41} + \frac{38}{41} = 2 + \frac{38}{41}.$$

The ten-thousandths digit is 2. *The difference between* $\frac{11}{41}$ *and* $\left(\frac{2}{10}+\frac{6}{100}+\frac{8}{1000}+\frac{2}{10000}\right)$ *is*

$$\frac{11}{41}-\left(\frac{2}{10}+\frac{6}{100}+\frac{8}{1000}+\frac{2}{10000}\right)=\frac{11}{41}-\frac{2682}{10000}=\frac{38}{410000}.$$

In the interval of hundred-thousandths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{100000}<\frac{38}{410000}<\frac{m+1}{100000},$$

which is the same as

$$m<\frac{380}{41}<m+1$$

$$\frac{380}{41}=\frac{369}{41}+\frac{11}{41}=9+\frac{11}{41}.$$

The hundred-thousandths digit is 9. *We see again the fraction* $\frac{11}{41}$, *so we can expect the decimal digits to repeat at this point. Therefore, the decimal approximation of* $\frac{11}{41}=0.2682926829\ldots$.

4. Use rational approximation to determine which number is larger, $\sqrt{10}$ or $\frac{28}{9}$.

The number $\sqrt{10}$ *is between* 3 *and* 4. *In the sequence of tenths,* $\sqrt{10}$ *is between* 3.1 *and* 3.2 *because* $3.1^2<\left(\sqrt{10}\right)^2<3.2^2$. *In the sequence of hundredths,* $\sqrt{10}$ *is between* 3.16 *and* 3.17 *because* $3.16^2<\left(\sqrt{10}\right)^2<3.17^2$. *In the sequence of hundredths,* $\sqrt{10}$ *is between* 3.162 *and* 3.163 *because* $3.162^2<\left(\sqrt{10}\right)^2<3.163^2$. *The decimal expansion of* $\sqrt{10}$ *is approximately* $3.162\ldots$.

$$\begin{aligned}\frac{28}{9}&=\frac{27}{9}+\frac{1}{9}\\&=3+\frac{1}{9}\end{aligned}$$

In the interval of tenths, we are looking for the integers m *and* $m+1$ *so that*

$$\frac{m}{10}<\frac{1}{9}<\frac{m+1}{10},$$

which is the same as

$$m<\frac{10}{9}<m+1$$

$$\begin{aligned}\frac{10}{9}&=\frac{9}{9}+\frac{1}{9}\\&=1+\frac{1}{9}.\end{aligned}$$

The tenths digit is 1. *Since the fraction* $\frac{1}{9}$ *has reappeared, then we can assume that the next digit is also* 1, *and the work will continue to repeat. Therefore, the decimal expansion of* $\frac{28}{9}=3.1111\ldots$.

Therefore, $\frac{28}{9}<\sqrt{10}$.

5. **Sam says that $\frac{7}{11} = 0.63$, and Jaylen says that $\frac{7}{11} = 0.636$. Who is correct? Why?**

In the interval of tenths, we are looking for integers m and $m+1$ so that

$$\frac{m}{10} < \frac{7}{11} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{70}{11} < \frac{m+1}{10}$$

$$\frac{70}{11} = \frac{66}{11} + \frac{4}{11}$$
$$= 6 + \frac{4}{11}.$$

The tenths digit is 6. The difference between $\frac{7}{11}$ and $\frac{6}{10}$ is

$$\frac{7}{11} - \frac{6}{10} = \frac{4}{110}.$$

In the interval of hundredths, we are looking for integers m and $m+1$ so that

$$\frac{m}{100} < \frac{4}{110} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{40}{11} < m+1$$
$$\frac{40}{11} = \frac{33}{11} + \frac{7}{11}$$
$$= 3 + \frac{7}{11}.$$

The hundredths digit is 3. Again, we see the fraction $\frac{7}{11}$, which means the next decimal digit will be 6, as it was in the tenths place. This means we will again see the fraction $\frac{4}{11}$, meaning we will have another digit of 3. Therefore, the decimal expansion of $\frac{7}{11}$ is 0.6363 ….

Then, technically, both Sam and Jaylen are incorrect because the fraction $\frac{7}{11}$ is an infinite decimal. However, Sam is correct to the first two decimal digits of the number, and Jaylen is correct to the first three decimal digits of the number.