Name ____ Date____________________

Linear Models

Suppose that a cell-phone monthly rate plan costs the user 5 cents per minute beyond a fixed monthly fee of $\$20$. This implies that the relationship between monthly cost and monthly number of minutes is linear.

1. Write an equation in words that relates total monthly cost to monthly minutes used. Explain how you found your answer.

2. Write an equation in symbols that relates the total monthly cost (y) to monthly minutes used (x).

3. What is the cost for a month in which 182 minutes are used? Express your answer in words in the context of this problem.

1. The Mathematics Club at your school is having a meeting. The advisor decides to bring bagels and his award-winning strawberry cream cheese. To determine his cost, from past experience he figures 1.5 bagels per student. A bagel costs 65 cents, and the special cream cheese costs $\$3.85$ and will be able to serve all of the anticipated students attending the meeting
   a. Find an equation that relates his total cost to the number of students he thinks will attend the meeting.
   b. In the context of the problem, interpret the slope of the equation in words.
   c. In the context of the problem, interpret the y-intercept of the equation in words. Does interpreting the intercept make sense? Explain.

2. John, Dawn, and Ron get together to exercise (walk/jog) for 45 minutes. John has arthritic knees but manages to walk $1\frac{1}{2}$ miles. Dawn walks $2\frac{1}{4}$ miles, while Ron manages to jog 6 miles.
   a. Draw an appropriate graph and connect the points to show that there is a linear relationship between the distance that each traveled based on how fast each traveled (speed). Note that the speed for a person who travels 3 miles in 45 minutes, or $\frac{3}{4}$ hours, is $3 \div \frac{3}{4} = 4$ miles per hour.
   b. Find an equation that expresses distance in terms of speed (how fast one goes).
   c. In the context of the problem, interpret the slope of the equation in words.
   d. In the context of the problem, interpret the y-intercept of the equation in words. Does interpreting the intercept make sense? Explain.

3. Simple interest is money that is paid on a loan. Simple interest is calculated by taking the amount of the loan and multiplying it by the rate of interest per year and the number of years the loan is outstanding. For college, Jodie's older brother has taken out a student loan for $\$4{,}500$ at an interest rate of 5.6%, or 0.056. When he graduates in four years, he will have to pay back the loan amount plus interest for four years. Jodie is curious as to how much her brother will have to pay.

   a. Jodie claims that his brother will have to pay her a total of $\$5{,}508$. Do you agree? Explain. As an example, 8% simple interest on $\$1{,}200$ for one year is $(0.08)(\$1200) = \$96$. The interest for two years would be $2 \times \$96$, or $\$192$.

   b. Write an equation for the total cost to repay a loan of $\$P$ if the rate of interest for a year is r (expressed as a decimal) for a time span of t years.

   c. If P and r are known, is the equation a linear equation?

   d. In the context of the problem, interpret the slope of the equation in words.

   e. In the context of the problem, interpret the intercept of the equation in words. Does interpreting the intercept make sense? Explain.

Suppose that a cell-phone monthly rate plan costs the user 5 cents per minute beyond a fixed monthly fee of $\$20$. This implies that the relationship between monthly cost and monthly number of minutes is linear.

1. **Write an equation in words that relates total monthly cost to monthly minutes used. Explain how you arrived at your answer.**

   The equation is given by total monthly cost $= 20 + 0.05$ (number of minutes used for a month).

   The y-intercept in the equation is the fixed monthly cost, $\$20$.

   The slope is the amount paid per minute of cell phone usage, or $\$0.05$ per minute.

   The linear form is total monthly cost $=$ fixed cost $+$ cost per minute (number of minutes used for a month).

2. **Write an equation in symbols that relates the total monthly cost (y) to monthly minutes used (x).**

   The equation is $y = 20 + 0.05x$, where y is the total cost for a month in dollars and x is cell phone usage for the month in minutes.

3. **What is the cost for a month in which 182 minutes are used? Express your answer in words in the context of this problem.**

   The total monthly cost in a month using 182 minutes would be
   20 dollars $+$ (0.05 dollars per minute)(182 minutes) $= \$29.10$.

   Be sure students pay attention to the meanings of the units, noting that units on one side of the equation must be the same as units on the other side.

1. **The Mathematics Club at your school is having a meeting. The advisor decides to bring bagels and his award-winning strawberry cream cheese. To determine his cost, from past experience he figures 1.5 bagels per student. A bagel costs 65 cents, and the special cream cheese costs $\$3.85$ and will be able to serve all of the anticipated students attending the meeting.**

   a. **Find an equation that relates his total cost to the number of students he thinks will attend the meeting.**

      Encourage students to write a problem in words in its context. For example, the advisor's total cost $=$ cream cheese fixed cost $+$ cost of bagels. The cost of bagels depends on the unit cost of a bagel times the number of bagels per student times the number of students. So, with symbols, if c denotes the total cost in dollars and n denotes the number of students, then $c = 3.85 + (0.65)(1.5)(n)$, or $c = 3.85 + 0.975n$.

   b. **In the context of the problem, interpret the slope of the equation in words.**

      For each additional student, the cost goes up by 0.975 dollars, or 97.5 cents.

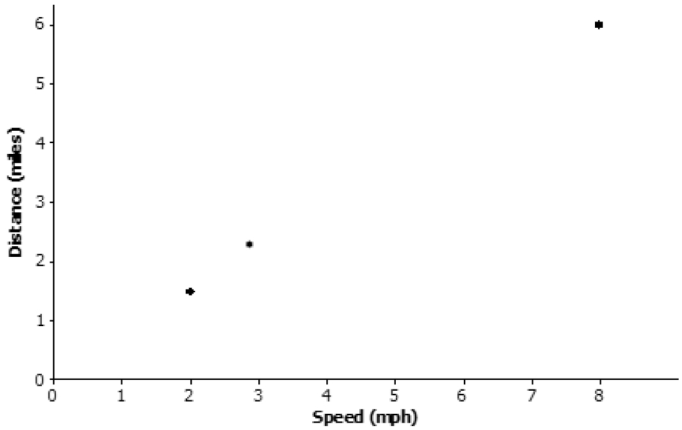
   c. **In the context of the problem, interpret the y-intercept of the equation in words. Does interpreting the intercept make sense? Explain.**

      If there are no students, the total cost is $\$3.85$. Students could interpret this by saying that the meeting was called off before any bagels were bought, but the advisor had already made his award-winning cream cheese, so the cost is $\$3.85$. The intercept makes sense. Other students might argue otherwise.

2. **John, Dawn, and Ron, get together to exercise (walk/jog) for 45 minutes. John has arthritic knees but manages to walk $1\frac{1}{2}$ miles. Dawn walks $2\frac{1}{4}$ miles, while Ron manages to jog 6 miles.**

   a. **Draw an appropriate graph and connect the points to show that there is a linear relationship between the distance that each traveled based on how fast each traveled (speed). Note that the speed for a person who travels 3 miles in 45 minutes, or $\frac{3}{4}$ hours, is $3 \div \frac{3}{4} = 4$ miles per hour.**

   John's speed is $\left(1\frac{1}{2} \div \frac{3}{4}\right) = 2$ miles per hour, Dawn's speed is $2\frac{1}{4} \div \frac{3}{4} = 3$ miles per hour, and Ron's speed is $6 \div \frac{3}{4} = 8$ miles per hour. Students may draw the scatter plot incorrectly. Note that distance is to be expressed in terms of speed so that distance is the dependent variable on the vertical axis, and speed is the independent variable on the horizontal axis.



   b. **Find an equation that expresses distance in terms of speed (how fast one goes).**

   The slope is $\frac{6-1.5}{8-2} = 0.75$, so the equation of the line through these points is distance $= a + (0.75)$(speed).

   Next, find the intercept. For example, solve $6 = a + (0.75)(8)$ for a, which yields $a = 0$.

   So, the equation is distance $= 0.75$(speed).

   c. **In the context of the problem, interpret the slope of the equation in words.**

   If someone increases his or her speed by 1 mile per hour, then that person travels 0.75 additional miles in 45 minutes.

   d. **In the context of the problem, interpret the y-intercept of the equation in words. Does interpreting the intercept make sense? Explain.**

   The intercept of 0 makes sense because if the speed is 0 miles per hour, then the person is not moving. So, the person travels no distance.

Note: Simple interest is developed in the next two problems. It is an excellent example of an application of a linear function. If students have not worked previously with finance problems of this type, then you may need to carefully explain simple interest as stated in the problem. It is an important discussion to have with students if time permits. If this discussion is not possible and students have not worked previously with any financial applications, then omit these problems.

3. **Simple interest is money that is paid on a loan. Simple interest is calculated by taking the amount of the loan and multiplying it by the rate of interest per year and the number of years the loan is outstanding. For college, Jodie's older brother has taken out a student loan for $\$4,500$ at an interest rate of 5.6%, or 0.056. When he graduates in four years, he will have to pay back the loan amount plus interest for four years. Jodie is curious as to how much her brother will have to pay.**

   a. **Jodie claims that his brother will have to pay a total of $\$5,508$. Do you agree? Explain. As an example, 8% simple interest on $\$1,200$ for one year is $(0.08)(1200) = \$96$. The interest for two years would be $2 \times \$96$, or $\$192$.**

   The total cost to repay $=$ amount of loan $+$ interest on the loan.

   Interest on the loan is the amount of simple interest for one year times the number of years the loan is outstanding.

   The annual simple interest amount is $(0.056)(\$4500) = \$252$ per year.

   For four years, the interest amount is $4(\$252) = \$1008$.

   So, the total cost to repay the loan is $\$4500 + \$1008 = \$5508$. Jodie is right.

   b. **Write an equation for the total cost to repay a loan of $\$P$ if the rate of interest for a year is r (expressed as a decimal) for a time span of t years.**

   Note: Work with students in identifying variables to represent the values discussed in this exercise. For example, the total cost to repay a loan is $P +$ the amount of interest on P for t years, or $P + I$, where $I =$ interest.

   The amount of interest per year is P times the annual interest. Let r represent the interest rate per year as a decimal.

   The total amount of simple interest for t years is rt, where r is the annual rate as a decimal (e.g., 5% is 0.05).

   So, if c denotes the total cost to repay the loan, then $c = P + (rt)P$.

   c. **If P and r are known, is the equation a linear equation?**

   If P and r are known, then the equation should be written as $c = P + (rP)t$, which is the linear form where c is the dependent variable and t is the independent variable.

   d. **In the context of this problem, interpret the slope of the equation in words.**

   For each additional year that the loan is outstanding, the total cost to repay the loan is increased by $\$rP$.

   As an example, consider Jodie's brother's equation for t years: $c = 4500 + (0.056)(4500)t$, or $c = 4500 + 252t$. For each additional year that the loan is not paid off, the total cost increases by $\$252$.

   e. **In the context of this problem, interpret the intercept of the equation in words. Does interpreting the intercept make sense? Explain.**

   The 0 value of time t means at the time the loan was taken out. At that time, no interest has been accumulated, so the intercept of $\$4,500$ as the cost to repay the loan after 0 years makes sense.