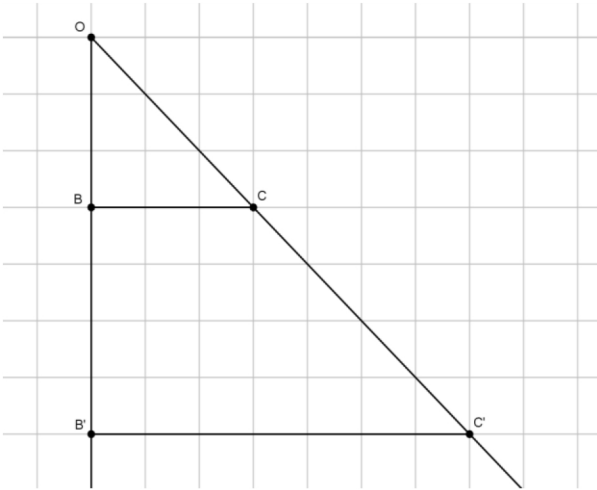


# Fundamental Theorem of Similarity (FTS)

Steven sketched the following diagram on graph paper. He dilated points  $B$  and  $C$  from point  $O$ . Answer the following questions based on his drawing.

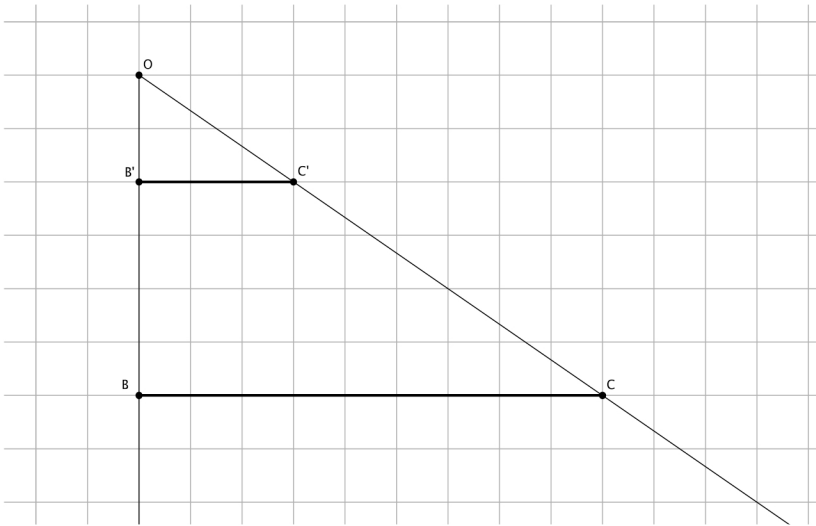
- 1. What is the scale factor  $r$ ? Show your work.
- 2. Verify the scale factor with a different set of segments.
- 3. Which segments are parallel? How do you know?
- 4. Are  $\angle OBC$  and  $\angle OB'C'$  right angles? How do you know?



1. Use a piece of notebook paper to verify the Fundamental Theorem of Similarity for a scale factor  $r$  that is  $0 < r < 1$ .

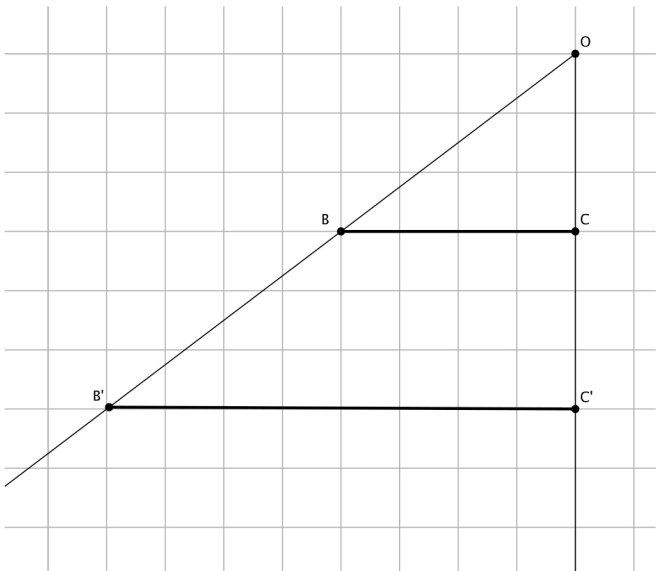
- ✓ Mark a point  $O$  on the first line of notebook paper.
  - ✓ Mark the point  $P$  on a line several lines down from the center  $O$ . Draw a ray,  $\overrightarrow{OP}$ . Mark the point  $P'$  on the ray, and on a line of the notebook paper, closer to  $O$  than you placed point  $P$ . This ensures that you have a scale factor that is  $0 < r < 1$ . Write your scale factor at the top of the notebook paper.
  - ✓ Draw another ray,  $\overrightarrow{OQ}$ , and mark the points  $Q$  and  $Q'$  according to your scale factor.
  - ✓ Connect points  $P$  and  $Q$ . Then, connect points  $P'$  and  $Q'$ .
  - ✓ Place a point  $A$  on line  $PQ$  between points  $P$  and  $Q$ . Draw ray  $\overrightarrow{OA}$ . Mark the point  $A'$  at the intersection of line  $P'Q'$  and ray  $\overrightarrow{OA}$ .
- a. Are lines  $PQ$  and  $P'Q'$  parallel lines? How do you know?
- b. Which, if any, of the following pairs of angles are equal in measure? Explain.
- i.  $\angle OPQ$  and  $\angle OP'Q'$
  - ii.  $\angle OAQ$  and  $\angle OA'Q'$
  - iii.  $\angle OAP$  and  $\angle OA'P'$
  - iv.  $\angle OQP$  and  $\angle OQ'P'$
- c. Which, if any, of the following statements are true? Show your work to verify or dispute each statement.
- i.  $|OP'| = r|OP|$
  - ii.  $|OQ'| = r|OQ|$
  - iii.  $|P'A'| = r|PA|$
  - iv.  $|A'Q'| = r|AQ|$
- d. Do you believe that the Fundamental Theorem of Similarity (FTS) is true even when the scale factor is  $0 < r < 1$ . Explain.

2. Caleb sketched the following diagram on graph paper. He dilated points  $B$  and  $C$  from center  $O$ .



- a. What is the scale factor  $r$ ? Show your work.
- b. Verify the scale factor with a different set of segments.
- c. Which segments are parallel? How do you know?
- d. Which angles are equal in measure? How do you know?

3. Points  $B$  and  $C$  were dilated from center  $O$ .



- a. What is the scale factor  $r$ ? Show your work.
- b. If the length of  $|OB| = 5$ , what is the length of  $|OB'|$ ?
- c. How does the perimeter of triangle  $OBC$  compare to the perimeter of triangle  $OB'C'$ ?
- d. Did the perimeter of triangle  $OB'C' = r \times$  (perimeter of triangle  $OBC$ )? Explain.

Steven sketched the following diagram on graph paper. He dilated points  $B$  and  $C$  from point  $O$ . Answer the following questions based on his drawing.

- What is the scale factor  $r$ ? Show your work.

$$\begin{aligned} |OB'| &= r|OB| \\ 7 &= r \times 3 \\ \frac{7}{3} &= r \end{aligned}$$

- Verify the scale factor with a different set of segments.

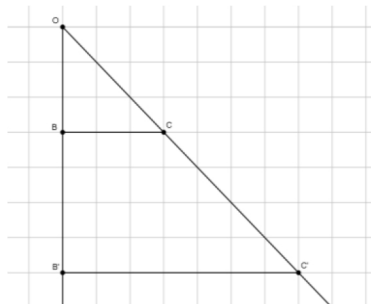
$$\begin{aligned} |B'C'| &= r|BC| \\ 7 &= r \times 3 \\ \frac{7}{3} &= r \end{aligned}$$

- Which segments are parallel? How do you know?

*Segments  $BC$  and  $B'C'$  are parallel since they lie on the grid lines of the paper, which are parallel.*

- Are  $\angle OBC$  and  $\angle OB'C'$  right angles? How do you know?

*The grid lines on graph paper are perpendicular, and since perpendicular lines form right angles,  $\angle OBC$  and  $\angle OB'C'$  are right angles.*

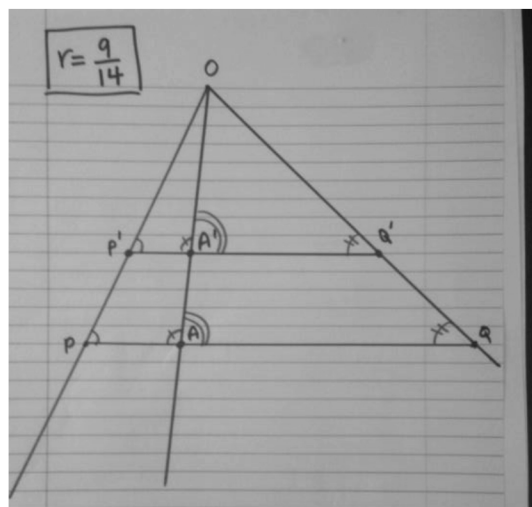


Students verify that the Fundamental Theorem of Similarity holds true when the scale factor  $r$  is  $0 < r < 1$ .

- Use a piece of notebook paper to verify the Fundamental Theorem of Similarity for a scale factor  $r$  that is  $0 < r < 1$ .

- ✓ Mark a point  $O$  on the first line of notebook paper.
- ✓ Mark the point  $P$  on a line several lines down from the center  $O$ . Draw a ray,  $\overrightarrow{OP}$ . Mark the point  $P'$  on the ray, and on a line of the notebook paper, closer to  $O$  than you placed point  $P$ . This ensures that you have a scale factor that is  $0 < r < 1$ . Write your scale factor at the top of the notebook paper.
- ✓ Draw another ray,  $\overrightarrow{OQ}$ , and mark the points  $Q$  and  $Q'$  according to your scale factor.
- ✓ Connect points  $P$  and  $Q$ . Then, connect points  $P'$  and  $Q'$ .
- ✓ Place a point  $A$  on line  $PQ$  between points  $P$  and  $Q$ . Draw ray  $\overrightarrow{OA}$ . Mark the point  $A'$  at the intersection of line  $P'Q'$  and ray  $\overrightarrow{OA}$ .

Sample student work shown in the picture below:



- a. Are lines  $PQ$  and  $P'Q'$  parallel lines? How do you know?

Yes, the lines  $PQ$  and  $P'Q'$  are parallel. The notebook lines are parallel, and these lines fall on the notebook lines.

- b. Which, if any, of the following pairs of angles are equal in measure? Explain.

- $\angle OPQ$  and  $\angle OP'Q'$
- $\angle OAQ$  and  $\angle OA'Q'$
- $\angle OAP$  and  $\angle OA'P'$
- $\angle OQP$  and  $\angle OQ'P'$

All four pairs of angles are equal in measure because each pair of angles are corresponding angles of parallel lines cut by a transversal. In each case, the parallel lines are line  $PQ$  and line  $P'Q'$ , and the transversal is the respective ray.

- c. Which, if any, of the following statements are true? Show your work to verify or dispute each statement.

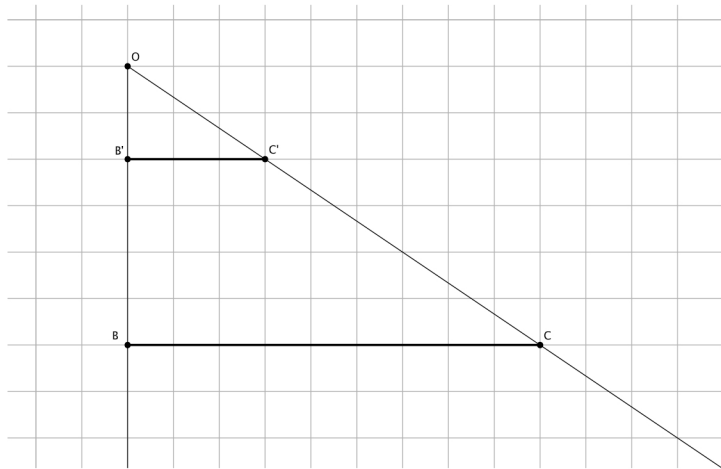
- $|OP'| = r|OP|$
- $|OQ'| = r|OQ|$
- $|P'A'| = r|PA|$
- $|A'Q'| = r|AQ|$

All four of the statements are true. Verify that students have shown that the length of the dilated segment was equal to the scale factor multiplied by the original segment length.

- d. Do you believe that the Fundamental Theorem of Similarity (FTS) is true even when the scale factor is  $0 < r < 1$ . Explain.

Yes, because I just experimentally verified the properties of FTS for when the scale factor is  $0 < r < 1$ .

2. Caleb sketched the following diagram on graph paper. He dilated points  $B$  and  $C$  from center  $O$ .



- a. What is the scale factor  $r$ ? Show your work.

$$|OB'| = r|OB|$$

$$2 = r \times 6$$

$$\frac{2}{6} = r$$

$$\frac{1}{3} = r$$

- b. Verify the scale factor with a different set of segments.

$$|B'C'| = r|BC|$$

$$3 = r \times 9$$

$$\frac{3}{9} = r$$

$$\frac{1}{3} = r$$

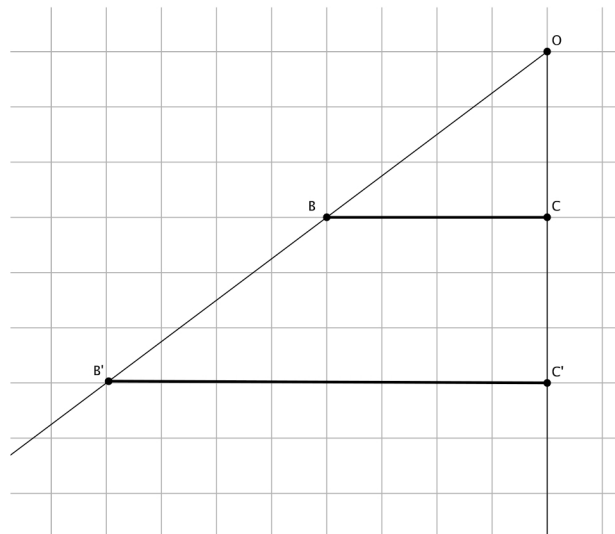
- c. Which segments are parallel? How do you know?

*Segment  $BC$  and  $B'C'$  are parallel. They lie on the lines of the graph paper, which are parallel.*

- d. Which angles are equal in measure? How do you know?

*$\angle OB'C' = \angle OBC$ , and  $\angle OC'B' = \angle OCB$  because they are corresponding angles of parallel lines cut by a transversal.*

3. Points  $B$  and  $C$  were dilated from center  $O$ .



- a. What is the scale factor  $r$ ? Show your work.

$$|OC'| = r|OC|$$

$$6 = r \times 3$$

$$\frac{6}{3} = r$$

$$2 = r$$

- b. If the length of  $|OB| = 5$ , what is the length of  $|OB'|$ ?

$$|OB'| = r|OB|$$

$$|OB'| = 2 \times 5$$

$$|OB'| = 10$$

- c. How does the perimeter of triangle  $OBC$  compare to the perimeter of triangle  $OB'C'$ ?

*The perimeter of triangle  $OBC$  is 12 units, and the perimeter of triangle  $OB'C'$  is 24 units.*

- d. Did the perimeter of triangle  $OB'C' = r \times$  (perimeter of triangle  $OBC$ )? Explain.

*Yes, the perimeter of triangle  $OB'C'$  was twice the perimeter of triangle  $OBC$ , which makes sense because the dilation increased the length of each segment by a scale factor of 2. That means that each side of triangle  $OB'C'$  was twice as long as each side of triangle  $OBC$ .*